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Computational Algebraic Geometry

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Preface

The theory and practice of computation in algebraic geometry and related domains, from a mathematical point of view, has generated an increasing interest both for its rich theoretical possibilities and its usefulness in applications in science and engineering. In fact, it is one of the master keys for future significant improvement of the computer algebra systems (e.g., Reduce, Macsyma, Maple, Mathematica, Axiom, Macaulay, etc.) that have become such useful tools for many scientists in a variety of disciplines.

The major themes covered in this volume, arising from papers presented at the conference MEGA-92 were:

- Effective methods and complexity issues in commutative algebra, projective geometry, real geometry, and algebraic number theory
- Algebro-geometric methods in algebraic computing and applications.

MEGA-92 was the second of a new series of European conferences on the general theme of Effective Methods in Algebraic Geometry. It was held in Nice, France, on April 21–25, 1992 and built on the themes presented at MEGA-90 (Livorno, Italy, April 17–21, 1990). The next conference — MEGA-94 — will be held in Santander, Spain in the spring of 1994. The Organizing committee that initiated and supervises this biennial conference consists of A. Conte (Torino), J. H. Davenport (Bath), A. Galligo (Nice), D. Yu. Grigoriev (Petersburg), J. Heintz (Buenos Aires), W. Lassner (Leipzig), D. Lazard (Paris), H. M. Möller (Hagen), T. Mora (Genova), M. Pohst (Düsseldorf), T. Recio (Santander), J. J. Risler (Paris), M. F. Roy (Rennes), R. Schoof (Utrecht), and C. Traverso (Pisa).

During the conference, an informal session was organized, in which some participants were given the opportunity to give short talks on current research, of too informal a nature for inclusion in the volume. In addition, the decision since was made to publish only in English, some excellent papers do not appear here. The following papers given at the conference were omitted from the volume:

Assi (Grenoble): Homogenization and standard bases with minimal ecart

Bjorck (Stockholm): There are $\infty + 1152$ cyclic 8-roots

Burgisser, Lickteig (Bonn): On verification complexity of linear prime ideals

De Concini (Roma): Deformations of quantum groups and representations

Gaeta (Madrid): Applications of associated forms (generalized chow forms) to elimination theory

Galligo (Nice), Vorobjov (Petersburg): Complexification of real algebraic varieties

Giusti (Paris), Heintz (Buenos-Aires): La détermination des points isolés et de la dimension d'une variété algébrique peut se faire en temps polynomial

Greuel (Kaiserslautern), Pfister (Berlin), Schoenemann (Kaiserslautern): The computer algebra system singular

Iarrobino (Boston), Yameogo (Nice): Partitions and ideals in $k[x, y]$

Jouanolou (Strasbourg): Formes d'inertie et applications

Labhalla (Marrakech), Lombardi (Besançon), Marlin (Nice): Computation of hermite normal form

Le Prevost (Paris): Famille de courbes hyperelliptiques de genre g munies d'une classe de diviseurs rationnels d'ordre $2g^2 + 4g + 1$

Mazurowskii (Ivanovo): Kauffman polynomial of non singular configurations of lines in RP^3

Mora (Genova): The complexity of the tangent cone algorithm

Morain (Paris): Primality proving using elliptic curves

Traverso et al. (Pisa): Natural representation of algebraic numbers

Van Effelterre (Leuven): Aspect graphs of solids of revolution

Yakoubsohn (Toulouse): The sturm theorem in the complex case

We wish to express our appreciation to these people whose presentations, though not included in the volume, made valuable contributions to the spirit and content of the conference.

To the organizers of MEGA-92 and the 114 participants, the conference was a gratifying success. The sessions were held in the renovated building of the Old Seminary at the seaside. The weather was excellent and the working atmosphere warm and stimulating. We wish to thank the members of the program committee and the referees whose work made the conference and the volume possible. A special votes of thanks goes to Odile Goëpp whose efficient cooperation made everything run more easily.

As a word of epilogue, an indirect consequence of the two symposia, MEGA-90 and MEGA-92, was the submission by a group of ten Euro-

pean teams led by members of our program committee of an ESPIRIT — Basic Research Action proposal called POSSO (Polynomial Systems Solving) to the European Community. The project is designed to provide realization in practice (including computer programmed aspects) of some mathematical ideas developed in MEGA and similar conferences. In the face of severe competition, the funding was offered and an official organization meeting held after MEGA-92 to plan MEGA-94 and beyond.

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