



Progress in Nonlinear Differential Equations and Their Applications

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Topological Nonlinear Analysis

Degree, Singularity, and Variations

Michele Matzeu
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Preface

Topological tools in Nonlinear Analysis had a tremendous development during the last few decades. The three main streams of research in this field, Topological Degree, Singularity Theory and Variational Methods, have lately become impetuous rivers of scientific investigation. The process is still going on and the achievements in this area are spectacular.

A most promising and rapidly developing field of research is the study of the role that symmetries play in nonlinear problems. Symmetries appear in a quite natural way in many problems in physics and in differential or symplectic geometry, such as closed orbits for autonomous Hamiltonian systems, configurations of symmetric elastic plates under pressure, Hopf Bifurcation, Taylor vortices, convective motions of fluids, oscillations of chemical reactions, etc... Some of these problems have been tackled recently by different techniques using equivariant versions of Degree, Singularity and Variations.

The main purpose of the present volume is to give a survey of some of the most significant achievements obtained by topological methods in Nonlinear Analysis during the last two-three decades. The survey articles presented here reflect the personal taste and points of view of the authors (all of them well-known and distinguished specialists in their own fields) on the subject matter. A common feature of these papers is that of starting with an historical introductory background of the different disciplines under consideration and climbing up to the heights of the most recent results. As a consequence, we obtain a very dynamic picture of the state of affairs. Actually, we hope to be able in the near future to involve other distinguished specialists to get their own versions on these topics. Most probably a never-ending fascinating tale!

Finally let us mention the fact that most of the material of this book was presented by the authors at the Topological Analysis Workshop on Degree, Singularity and Variations, held in May 1993 at Villa Campitelli, Frascati, near Rome.

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