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Control of Uncertain Sampled-Data Systems

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To my parents

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Preface

My main goal in writing this monograph is to provide a detailed treatment of uncertainty analysis for sampled-data systems in the context of systems control theory. Here, sampled-data system refers to the *hybrid* system formed when continuous time and discrete time systems are interconnected; by uncertainty analysis I mean achievable performance in the presence of worst-case uncertainty and disturbances. The focus of the book is sampled-data systems; however the approach presented is applicable to both standard and sampled-data systems.

The past few years has seen a large surge in research activity centered around creating systematic methods for sampled-data design. The aim of this activity has been to deepen and broaden the, by now, sophisticated viewpoint developed for design of purely continuous time or discrete time systems (e.g. $\mathcal{H}_\infty$ or ℓ_1 optimal synthesis, μ theory) so that it can be applied to the design of sampled-data systems. This research effort has been largely successful, producing both interesting new mathematical tools for control theory, and new methodologies for practical engineering design.

Analysis of *structured* uncertainty is an important objective in control design, because it is a flexible and non-conservative way of analyzing system performance, which is suitable in many engineering design scenarios. For this reason it has been studied extensively in a number of mathematical frameworks since the advent of the structured singular value in the early eighties. In this book both analysis techniques and results are developed to address the main sampled-data analysis problems in the context of structured uncertainty. The impact of several related classes of structured dynamic uncertainty on system performance is studied. The resulting development is of both engineering and theoretical interest, and I have endeavored to give a complete picture of which of the book's results are similar to those for purely continuous time systems, and which are surprising and unique to sampled-data systems.

The theory is developed primarily in an operator theoretic framework, and specifically the signal space of interest is $\mathcal{L}_2$. Although all the required background mathematics is collected in Chapter 2, having some familiarity with operators on Hilbert space and frequency domain methods will be a definite asset.

The bulk of this research was carried out from October 1990 to January 1994 while I was a graduate student at Cambridge University. I gratefully acknowledge my sources of financial support during this period which were Peterhouse and the Science and Engineering Research Council (UK). I also acknowledge my current financial support, while a Research Fellow at Caltech, which is provided by the Air Force Office of Scientific Research (USA).

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Pasadena, California
August, 1995

Geir E. Dullerud

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Notation

For the purposes of reference, the following glossary outlines the notation and frequently used objects in the text.

$\mathbb{Z}_+$	the non-negative integers
$\mathbb{R}$, $\mathbb{R}_+$	real and non-negative real numbers
$\mathbb{C}$, $\mathbb{C}_+$, $\bar{\mathbb{C}}_+$	complex number, open right half-plane, closed right half-plane
$\mathbb{D}$, $\bar{\mathbb{D}}$, $\partial\mathbb{D}$	open unit disc, closed unit disc, unit circle
$\cap$	set intersection
$\times$	Cartesian product
$\oplus$	orthogonal direct sum
$\lvert\cdot\rvert_2$, $\langle\cdot,\cdot\rangle$	Euclidean norm and inner product
$*$	conjugate or adjoint
$\lVert\cdot\rVert_{\mathcal{H}}$, $\langle\cdot,\cdot\rangle_{\mathcal{H}}$	norm and inner product on $\mathcal{H}$
$\lVert\cdot\rVert_{\mathcal{H}_1\to\mathcal{H}_2}$	induced $\mathcal{H}_1$ to $\mathcal{H}_2$ operator norm
$\mathfrak{L}(\mathcal{H}_1, \mathcal{H}_2)$	bounded linear operators from $\mathcal{H}_1$ to $\mathcal{H}_2$
$\mathfrak{C}(\mathcal{H}_1, \mathcal{H}_2)$	compact operators from $\mathcal{H}_1$ to $\mathcal{H}_2$
$\mathcal{U}\mathcal{W}$	open unit ball of the normed space $\mathcal{W}$
$\mathrm{spec}(\cdot)$	spectrum
$\mathrm{rad}(\cdot)$	spectral radius
$\to$	maps to, tends to
$:=$	right hand side defines the left hand side
Im	image of operator
ker	kernel of operator
$\mathcal{H}^{\perp}$	orthogonal complement of the subspace $\mathcal{H}$
$\bar{\sigma}(\cdot)$	maximum singular value
$\lambda_{\max}(\cdot)$	maximum eigenvalue
$\mathcal{F}_u(\cdot,\cdot)$, $\mathcal{F}_l(\cdot,\cdot)$	upper and lower linear fractional transformations
$\mathrm{diag}(\cdot)$	diagonal operator with given entries
$\mu_{\mathbf{\Delta}}(\cdot)$	structured singular value with respect to the set $\mathbf{\Delta}$
RHS, LHS	right hand and left hand sides

LTI	linear time-invariant
LMI	linear matrix inequality
$\mathcal{L}_2^m$	square integrable functions mapping $[0, \infty)$ to $\mathbb{C}^m$
$\mathcal{K}_2^m$	square integrable functions from $[0, h)$ to $\mathbb{C}^m$
ℓ_2^m	square summable sequences mapping $\mathbb{Z}_+$ to $\mathbb{C}^m$
$\boldsymbol{\ell}_2^m$	square summable sequences mapping $\mathbb{Z}_+$ to $\mathcal{K}_2^m$
$\boldsymbol{\mathcal{H}}_2^m$	square integrable functions from $\mathbb{D}$ to $\mathcal{K}_2^m$
$\boldsymbol{\mathcal{H}}_\infty$, $\boldsymbol{\mathcal{A}}$	spaces of operator valued analytic functions, Section 2.5
$\mathcal{H}_\infty$, $\mathcal{A}_\mathbb{R}$	spaces of matrix valued analytic functions, Section 2.5
$\mathfrak{L}_{\mathcal{A}_\mathbb{R}}$	LTI operators on $\mathcal{L}_2$ defined by $\mathcal{A}_\mathbb{R}$
$\mathfrak{L}_{\boldsymbol{\mathcal{A}}}$	periodic operators on $\mathcal{L}_2$ defined by $\boldsymbol{\mathcal{A}}$
$\boldsymbol{\mathcal{A}}_{\mathcal{A}_\mathbb{R}}$	image of the functions $\mathcal{A}_\mathbb{R}$ in $\boldsymbol{\mathcal{A}}$
$\mathfrak{X}_s$	spatially structured operators on $\mathcal{L}_2$, Chapter 3
$\mathcal{X}$	structured set of matrices, Chapter 4
$\mathcal{A}_\mathbb{R}^{\mathcal{X}}$	subclass of $\mathcal{A}_\mathbb{R}$ mapping to $\mathcal{X}$
$\mathfrak{X}_{LTI}$	LTI operators in $\mathfrak{X}_s$
$\mathfrak{X}_{PTV}$	intersection of $\mathfrak{X}_s$ and the periodic operators $\mathfrak{L}_{\boldsymbol{\mathcal{A}}}$
$\boldsymbol{\Delta}_{LTI}$	structured set of operators on ℓ_2, Section 4.3
$\boldsymbol{\Delta}_{PTV}$	structured set of operators on $\mathcal{K}_2$, Section 6.1
h	sampling period
$\mathbf{S}$	sampling operator
$\mathbf{H}$	hold operator
W	sampled-data lifting operator, Section 2.4
Z	the z-transform, Section 2.5
$\tilde{U}$	the unilateral shift on $\boldsymbol{\ell}_2$
$\mathbf{D}_h$	the h delay on $\mathcal{L}_2$
ν_k	the sequence $\{0, 1, -1, 2, -2, \ldots\}$