Applied and Numerical Harmonic Analysis

Gabor Analysis and Algorithms

Applied and Numerical Harmonic Analysis

Series Editor
John J. Benedetto
University of Maryland

Gabor Analysis and Algorithms

Theory and Applications

Hans G. Feichtinger
Thomas Strohmer
Editors

Springer Science+Business Media, LLC

Hans G. Feichtinger
Thomas Strohmer
Department of Mathematics
University of Vienna
Vienna A-1090
Austria

Library of Congress Cataloging-in-Publication Data

Gabor analysis and algorithms : theory and applications / edited by
Hans G. Feichtinger, Thomas Strohmer.
p. cm. -- (Applied and numerical harmonic analysis)
Includes bibliographical references and index.
ISBN 978-1-4612-7382-0 ISBN 978-1-4612-2016-9 (eBook)
DOI 10.1007/978-1-4612-2016-9
1. Signal processing--Digital techniques--Mathematics. 2. Image processing--Digitial techniques--Mathematics. 3. Algorithms.
I. Feichtinger, Hans G., 1951- . II. Strohmer, Thomas, 1965-
TK5102.9.G33 1997
621.382'2'015152433--dc21 97-32252
CIP

Printed on acid-free paper

ISBN 978-1-4612-7382-0

Typeset by the Editors in LaTeX2E.
Cover design by Benchmark Productions, Inc., Boston, MA.

9 8 7 6 5 4 3 2 1

Contents

Foreword

In his paper *Theory of Communication* [Gab46], D. Gabor proposed the use of a family of functions obtained from one Gaussian by time- and frequency-shifts. Each of these is well concentrated in time and frequency; together they are meant to constitute a complete collection of building blocks into which more complicated time-depending functions can be decomposed. The application to communication proposed by Gabor was to send the coefficients of the decomposition into this family of a signal, rather than the signal itself. This remained a proposal—as far as I know there were no serious attempts to implement it for communication purposes in practice, and in fact, at the critical time-frequency density proposed originally, there is a mathematical obstruction; as was understood later, the family of shifted and modulated Gaussians spans the space of square integrable functions [BBGK71, Per71] (it even has one function to spare [BGZ75] ...) but it does not constitute what we now call a *frame*, leading to numerical instabilities. The Balian-Low theorem (about which the reader can find more in some of the contributions in this book) and its extensions showed that a similar mishap occurs if the Gaussian is replaced by any other function that is "reasonably" smooth and localized. One is thus led naturally to considering a higher time-frequency density.

Interestingly, the same time-frequency lattice of functions was also proposed in an entirely different context by von Neumann [vN55], and became subsequently known as the von Neumann lattice, and lived an essential parallel life among quantum physicists (witness [BBGK71, Per71, BGZ75]). In addition, there is also a very clear connection to the short-time Fourier transform or windowed Fourier transform, used extensively in electrical engineering. Here too, the need to go to *overcritical sampling*, corresponding to the higher time-frequency density mentioned above, was discovered, independently.

Of course, in order to be useful practically, a transform must not only have good mathematical properties; it must also go hand-in-hand with efficient discrete algorithms, and for the Gabor transform these were developed extensively in the last decade.

Yet, despite this long history, and a lot of work by mathematicians, physicists and engineers alike, there are still many interesting and useful aspects of the Gabor transform to be explored and exploited. This book is an illustration of the continuing vigor of research on the Gabor transform, with mathematical developments, as well as approaches to numerical algorithms and a variety of applications.

Ingrid Daubechies
Princeton, New Jersey

Preface

This is the first book devoted to the subject of *Gabor analysis*. Since Dennis Gabor's fundamental paper of 1946, half a century has passed, but only in the last 10–15 years Gabor expansions have gained popularity in the signal processing community and under mathematicians. A number of basic questions has been put on firm mathematical grounds, and on the practical side efficient algorithms for numerical implementations have been developed, not to mention the variety of applications presented over the years.

The editors have asked a team of authors to cover the wide range of problems and methods coming together in Gabor analysis, and to give readers a survey of the present state of the field. We believe that the field has reached a first stage of maturity, which suggests summarizing existing results, trying to unify terminology and prepare ground for further investigations.

In this sense we also anticipate hope that this book will become a widely used general reference, and that it will motivate further research in the field and stimulate communication between mathematicians, engineers, and other scientists. We also hope to demonstrate through this book that Gabor analysis is not just "the unimportant, old-fashioned uncle" of the wavelet transform, but a fascinating field of mathematics and signal analysis, still offering high potential for further applications.

The book is addressing a broad audience, such as mathematicians looking for interesting problems with relevance to signal processing, as well as the engineering community or computer scientists, who care for efficient algorithms, and to applied scientists looking for powerful methods of signal analysis. The book is also supposed to provide rich material for graduate seminars and courses on Gabor analysis.

Due to the diversity of topics covered in the different chapters, the required background to fully appreciate their content varies. Because this book is written by scientists from different fields, most readers will find it appropriate to start with the topic of their main interest, and collect relevant (mostly mathematical) background by following the cross connections to other chapters. The book is sufficiently self-contained in order to allow a reader with a general background in signal analysis (or alternatively just the corresponding concepts of mathematical analysis) to profit from it and to use it as a guide toward a better understanding of Gabor analysis.

We encourage all readers, in particular those who would like to send some constructive criticism, to contact the editors by email or through the *Gabor Digest*, located at `http://tyche.mat.univie.ac.at`. This forum may be used by the readers of this book to find out about comments on the articles, updates, hints to recent publications, and further valuable information.

Acknowledgements

We would like to thank the authors for their excellent contributions and their willingness to follow our editorial suggestions in order to give the book a more uniform appearance. We gratefully acknowledge stimulating discussions with Karlheinz Gröchenig and generous assistance of Johann "Niki" Lutz, Norbert Kaiblinger, Peter Prinz, Mitch Rauth, and Georg Zimmermann from the Numerical Harmonic Analysis Group (NUHAG) at our Department. In particular we are greatly indebted to Werner Kozek (also from the NUHAG team) for his constant support and valuable suggestions. We are grateful to Lauren Lavery and Wayne Yuhasz of Birkhäuser for their friendly help, and to John Benedetto, the editor of the Applied and Numerical Harmonic Analysis series, who encouraged us to edit this book. Finally we want to thank Katharina and Gabriela for their understanding and support during the period of preparation for this book.

Hans G. Feichtinger and Thomas Strohmer
Vienna, Austria

Contributors

Martin J. Bastiaans Department of Electrical Engineering, University of Technology, Eindhoven, Netherlands, [*m.j.bastiaans@ele.tue.nl*]

Jezekiel Ben-Arie Electrical Engineering and Computer Science Department, University of Illinois, Chicago, Illinois, [*benarie@eecs.uic.edu*]

John J. Benedetto Department of Mathematics, University of Maryland, College Park, Maryland and The MITRE Corporation, McLean, Virginia, [*jjb@math.umd.edu*]

Helmut Bölcskei Department of Communications and Radio-Frequency Engineering, Vienna University of Technology, Vienna, Austria, [*hboelcsk@aurora.nt.tuwien.ac.at*]

Ole Christensen Department of Mathematics, Technical University of Denmark, Lyngby, Denmark, [*olechr@mat.dtu.dk*]

Hans G. Feichtinger Department of Mathematics, University of Vienna, Vienna, Austria, [*fei@tyche.mat.univie.ac.at*]

Benjamin Friedlander Department of Electrical and Computer Engineering, University of California, Davis, California, [*friedlan@ece.ucdavis.edu*]

Karlheinz Gröchenig Department of Mathematics, University of Connectiicut, Storrs, Connecticut, [*groch@math.uconn.edu*]

Christopher Heil School of Mathematics, Georgia Institute of Technology, Atlanta, Georgia and The MITRE Corporation, Bedford, Massachusetts, [*heil@math.gatech.edu*]

Franz Hlawatsch Department of Communications and Radio-Frequency Engineering, Vienna University of Technology, Vienna, Austria, [*fhlawats@email.tuwien.ac.at*]

A.J.E.M. Janssen Philips Research Laboratories, Eindhoven, The Netherlands, [*janssena@natlab.research.philips.com*]

Werner Kozek Department of Mathematics, University of Vienna, Vienna, Austria, [*kozek@tyche.mat.univie.ac.at*]

Moshe Porat Department of Electrical Engineering, Technion – Israel Institute of Technology, Haifa, Israel, [*mp@ee.technion.ac.il*]

Richard Rochberg Department of Mathematics, Washington University, St. Louis, Missouri, [*rr@math.wustl.edu*]

Thomas Strohmer Department of Mathematics, University of Vienna, Vienna, Austria, [*strohmer@tyche.mat.univie.ac.at*]

Kazuya Tachizawa Mathematical Institute, Tohoku University, Sendai, Japan, [*tachizaw@math.tohoku.ac.jp*]

David F. Walnut Department of Mathematical Sciences, George Mason University, Fairfax, Virginia, [*dwalnut@gmu.edu*]

Zhiqian Wang Electrical Engineering and Computer Science Department, University of Illinois, Chicago, Illinois, [*zwang@eecs.uic.edu*]

Yehoshua Y. Zeevi Department of Electrical Engineering, Technion – Israel Institute of Technology, Haifa, Israel, [*zeevi@ee.technion.ac.il*]

Ariela Zeira Signal Processing Technology, Palo Alto, California, [*Zeira@compuserve.com*]

Meir Zibulski IBM Science & Technology, Haifa, Israel, [*meirz@vnet.ibm.com*]

Georg Zimmermann Department of Mathematics, University of Vienna, Vienna, Austria, [*gzim@tyche.mat.univie.ac.at*]