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Avner Friedman   Robert Gulliver

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Continued at the back

Krishna B. Athreya      Peter Jagers  
Editors

# Classical and Modern Branching Processes

With 22 Illustrations



Springer

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## **FOREWORD**

This IMA Volume in Mathematics and its Applications

### **CLASSICAL AND MODERN BRANCHING PROCESSES**

is based on the proceedings with the same title and was an integral part of the 1993–94 IMA program on “Emerging Applications of Probability.” We would like to thank Krishna B. Athreya and Peter Jagers for their hard work in organizing this meeting and in editing the proceedings. We also take this opportunity to thank the National Science Foundation, the Army Research Office, and the National Security Agency, whose financial support made this workshop possible.

Avner Friedman

Robert Gulliver

## PREFACE

The IMA workshop on Classical and Modern Branching Processes was held during June 13–17 1994 as part of the IMA year on Emerging Applications of Probability.

The organizers of the year long program identified branching processes as one of the active areas in which a workshop should be held. Krishna B. Athreya and Peter Jagers were asked to organize this.

The topics covered by the workshop could broadly be divided into the following areas:

1. Tree structures and branching processes;
2. Branching random walks;
3. Measure valued branching processes;
4. Branching with dependence;
5. Large deviations in branching processes;
6. Classical branching processes.

The breadth and depth of the papers presented at the workshop showed the vitality of the subject, the rich scope for new work, and the relevance of branching ideas in the study of topics ranging from other parts of mathematics to computer science, physics, and of course biology.

On Topic 1, Tree structures in branching processes, Lyons *et al.* present a new approach to the famous LLOGL theorem of Kesten and Stigum for multi-type processes. This approach is more measure theoretic and less analytic. Lyons uses it also to prove Biggins' theorem for branching random walks. (Since the workshop, Olofsson has used these ideas to cover the CMJ process and Athreya has a version for general branching.) Lyons *et al.* in a second paper list a number of unsolved problems on random walks on trees. Pemantle shows the sharpness of the second moment condition for tree indexed processes. Liu and Roualt show the singularity of two measures on the boundary of a Galton-Watson process. Geiger and Kersting study the Markov structure of contours of certain trees.

Topic 2, branching random walks is a well-studied area in which a number of new and exciting contributions were made at the workshop. Biggins shed more light on the old problem of the right-most particle and applied it to data storage algorithms. Dekking and Speer study the left-most particle in a branching random walk with bounded steps and also the associated wavefront. Suhov and Karpelevich study the absolute maximum of a branching random walk. The following papers involve the application of branching random walks in statistical physics. Chauvin and Roualt establish the almost-sure convergence of the free energy of disordered systems. Koukiou treats the mean-field theory for spin glasses and directed polymers. Waymire extends the Kahane-Peyriere theory of independent cascades to finite-state Markov ones.

Topic 3, Measure-valued branching processes (precursors of Dynkin's superprocesses) has contributions from Dawson and Wu who study a class of multilevel branching particle systems and apply their results to a model for dynamical information systems, from Lopez-Mimbela and Wakolbinger on multitype branching populations that move in some space where they study the survival question, and from Sagitov who analyzes the limit structure of subtrees with many children in critical CMJ processes.

Topic 4 deals with branching processes where the key assumption of independence of lines of descent is dropped. Olofsson treats sibling dependent offspring production. Jagers discusses local and related dependence in a general formulation. Kimmel considers a biological application. Klebaner treats population dependent offspring production in near-critical Galton-Watson processes.

Topic 5 deals with large-deviation problems in supercritical and critical branching processes. Athreya and Vidyashankar show that in multi-type cases the large deviation of certain empirical averaged decay geometrically and under appropriate conditioning super-geometrically. Biggins' paper also falls in this area.

Topic 6 focuses on classical branching processes. Cohn uses stochastic monotonicity to provide a unified treatment of simple branching processes. Dromta and Vatutin consider a special two-type process. Pakes gives a simple condition for a given sequence to be a harmonic renewal sequence. Rahimov studies record values in a family of branching processes. Yanev and Yanev develop some limit theory for processes with random immigration stopped at zero.

Not all the speakers at the workshop submitted contributions. All written contributions were refereed.

The organizers Krishna B. Athreya and Peter Jagers appreciate the help from all the participants in making the workshop a success. We are also very grateful to the IMA and its truly wonderful staff for all the help and hospitality during the two month period that ended with the workshop. Our special thanks go to Ms. Patricia V. Brick for her guiding us thru this editorial work. Finally we want to thank Avner Friedman and Robert Gulliver, and the committee that ran the year long program for asking us to organize the workshop and the constant help they gave us.

Krishna B. Athreya and Peter Jagers

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