# Applied Mathematical Sciences

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Vy Khoi Le  Klaus Schmitt

# Global Bifurcation in Variational Inequalities

## Applications to Obstacle and Unilateral Problems

With 22 Illustrations

Springer

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Kính tặng Bố Mẹ  To Claudia, Susan, and Michael

Angesichts von Hindernissen
mag der kürzeste Weg zwischen zwei Punkten
der krumme sein.

– *Bertolt Brecht*
*Leben des Galilei*

# Contents

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# List of Symbols

| | |
|---|---|
| $\mathbb{R}$ | The real line. |
| $\mathbb{R}^N$ | $N$-dimensional Euclidean space. |
| $\mathbb{R}_*^+$ | $= \{x \in \mathbb{R} : x > 0\}$, the set of positive real numbers. |
| $\Omega$ | An open bounded domain in $\mathbb{R}^N$. |
| $X$, $V$ | Real reflexive Banach spaces. |
| $V^*$ | The dual space of $V$. |
| $L(X, V)$ | The space of bounded linear mappings from $X$ to $V$. |
| $B_r(a)$ | $= \{x : \|x - a\| < r\}$, the ball of radius $r$, centered at $a$. |
| $\overline{K}$ | The closure of a subset $K$ of $V$. |
| $\overset{\circ}{K}$ | The interior part of $K$. |
| $\partial K$ | The boundary of $K$. |
| $\nabla f$ | $= (\partial f/\partial x_1, \partial f/\partial x_2, \ldots, \partial f/\partial x_N)$, the gradient of $f$. |
| $\Delta f$ | $= \partial^2 f/\partial x_1^2 + \partial^2 f/\partial x_2^2 + \cdots + \partial^2 f/\partial x_N^2$, the Laplacian of $f$. |
| $p'$ | $= p(p-1)^{-1}$, the conjugate exponent of $p$. |
| $p^*$ | $= Np(N-p)^{-1}$, the Sobolev conjugate exponent of $p$. |
| $\|f\|_{L^p(\Omega)}$ | $= \left(\int_\Omega \|f\|^p dx\right)^{1/p}$, the $L^p$ norm. |
| $L^p(\Omega)$ | The space of $p$ integrable functions (whose $L^p$ norm is bounded). |
| $\|f\|_{W^{k,p}(\Omega)}$ | $= \left(\sum_{\|\beta\| \le k} \int_\Omega \|D^\beta f\|^p dx\right)^{1/p}$, the Sobolev norm. |

| | |
|---|---|
| $W^{k,p}(\Omega)$ | The space of functions with bounded $W^{k,p}(\Omega)$ Sobolev norm. |
| $C_0^\infty(\Omega)$ | $C^\infty$ functions with compact support. |
| $W_0^{k,p}(\Omega)$ | The closure in $W^{k,p}(\Omega)$ of $C_0^\infty(\Omega)$. |
| $H^k(\Omega)$ | $= W^{k,2}(\Omega)$. |
| $H_0^k(\Omega)$ | $= W_0^{k,2}(\Omega)$. |
| $\mathrm{supp} f$ | The support of a function $f$. |