

Applied Mathematical Sciences

Volume 123

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Springer Science+Business Media, LLC

Applied Mathematical Sciences

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(continued following index)

Vy Khoi Le Klaus Schmitt

Global Bifurcation in Variational Inequalities

Applications to Obstacle and
Unilateral Problems

With 22 Illustrations



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Mathematics Subject Classification (1991): 73CS0, 73Kxx, 49Rxx, 73Hxx

Library of Congress Cataloging-in-Publication Data
Le, Vy Khoi

Global bifurcation in variational inequalities : applications to
obstacle and unilateral problems / Vy Khoi Le, Klaus Schmitt.

p. cm. -- (Applied Mathematical Sciences ; 123)

Includes bibliographic references and index.

ISBN 0-387-94886-4 (alk. paper)

1. Variational inequalities (Mathematics) 2. Bifurcation theory.

I. Schmitt, Klaus, 1940- . II. Title. III. Series: Applied
mathematical sciences (Springer-Verlag New York Inc.) ; v. 123.

QA1.A647 vol. 123

[QA316]

5110 s—dc20

[515'.36]

96-41133

Printed on acid-free paper.

© 1997 Springer Science+Business Media New York

Originally published by Springer-Verlag New York, Inc. in 1997

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Production managed by Robert Wexler; manufacturing supervised by Jacqui Ashri.
Photocomposed pages prepared from the authors' \TeX file.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4612-7298-4

ISBN 978-1-4612-1820-3 (eBook)

DOI 10.1007/978-1-4612-1820-3

Kính tặng Bố Mẹ To Claudia, Susan, and Michael

Angesichts von Hindernissen
mag der kürzeste Weg zwischen zwei Punkten
der krumme sein.

– *Bertolt Brecht*
Leben des Galilei

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List of Symbols

\mathbb{R}	The real line.
\mathbb{R}^N	N -dimensional Euclidean space.
\mathbb{R}_*^+	$= \{x \in \mathbb{R} : x > 0\}$, the set of positive real numbers.
Ω	An open bounded domain in \mathbb{R}^N .
X, V	Real reflexive Banach spaces.
V^*	The dual space of V .
$L(X, V)$	The space of bounded linear mappings from X to V .
$B_r(a)$	$= \{x : \ x - a\ < r\}$, the ball of radius r , centered at a .
\bar{K}	The closure of a subset K of V .
$\overset{\circ}{K}$	The interior part of K .
∂K	The boundary of K .
∇f	$= (\partial f / \partial x_1, \partial f / \partial x_2, \dots, \partial f / \partial x_N)$, the gradient of f .
Δf	$= \partial^2 f / \partial x_1^2 + \partial^2 f / \partial x_2^2 + \dots + \partial^2 f / \partial x_N^2$, the Laplacian of f .
p'	$= p(p - 1)^{-1}$, the conjugate exponent of p .
p^*	$= Np(N - p)^{-1}$, the Sobolev conjugate exponent of p .
$\ f\ _{L^p(\Omega)}$	$= (\int_{\Omega} f ^p dx)^{1/p}$, the L^p norm.
$L^p(\Omega)$	The space of p integrable functions (whose L^p norm is bounded).
$\ f\ _{W^{k,p}(\Omega)}$	$= \left(\sum_{ \beta \leq k} \int_{\Omega} D^{\beta} f ^p dx \right)^{1/p}$, the Sobolev norm.

$W^{k,p}(\Omega)$	The space of functions with bounded $W^{k,p}(\Omega)$ Sobolev norm.
$C_0^\infty(\Omega)$	C^∞ functions with compact support.
$W_0^{k,p}(\Omega)$	The closure in $W^{k,p}(\Omega)$ of $C_0^\infty(\Omega)$.
$H^k(\Omega)$	$= W^{k,2}(\Omega)$.
$H_0^k(\Omega)$	$= W_0^{k,2}(\Omega)$.
$\text{supp} f$	The support of a function f .