Modeling and Simulation in Science, Engineering and Technology

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Foundations of Fluid Mechanics with Applications

Problem Solving Using *Mathematica*®

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Preface

Fluid mechanics (FM) is a branch of science dealing with the investigation of flows of continua under the action of external forces. The fundamentals of FM were laid in the works of the famous scientists, such as L. Euler, M.V. Lomonosov, D. Bernoulli, J.L. Lagrange, A. Cauchy, L. Navier, S.D. Poisson, and other classics of science. Fluid mechanics underwent a rapid development during the past two centuries, and it now includes, along with the above branches, aerodynamics, hydrodynamics, rarefied gas dynamics, mechanics of multiphase and reactive media, etc.

The FM application domains were expanded, and new investigation methods were developed. Certain concepts introduced by the classics of science, however, are still of primary importance and will apparently be of importance in the future. The Lagrangian and Eulerian descriptions of a continuum, tensors of strains and stresses, conservation laws for mass, momentum, moment of momentum, and energy are the examples of such concepts and results. This list should be augmented by the first and second laws of thermodynamics, which determine the character and direction of processes at a given point of a continuum. The availability of the conservation laws is conditioned by the homogeneity and isotropicity properties of the Euclidean space, and the form of these laws is related to the Newton's laws. The laws of thermodynamics have their foundation in the statistical physics. These concepts and laws are insufficient for the description of continuum motion, however, they should be augmented by the equations of state or, as one says sometimes, by the closure relationships, which relate the tensors of strains and stresses to their derivatives. Each continuum possesses its own equations of state, which are found by processing the experimental data or by solving the corresponding problems from the statistical physics.

The concept of continuum does not contain in itself the information about the discrete character of substance at the molecular level, all this information is laid in thermodynamics laws and closure relationships (equations of state). It follows from here that the FM is no closed field of knowledge; it has a tight interaction with other branches of physics and mathematics.

The teaching of FM has its remarkable traditions laid in the lecture courses of L.D. Landau and E.M. Lifschitz, L.I. Sedov, N.I. Kochin, L. Prandtl, G. Batchelor, W. Prager, P. Germain, and other authors. New interesting results have appeared in FM after the publication of these lecture courses, however, and there is now a need in presenting them in the educational literature, so that students can be rapidly introduced into the scope of present-day FM problems and methods.

The development of computer algebra and of a powerful universal software system *Mathematica* has led to the fact that the task of the FM presentation with the use of the *Mathematica* system has become topical. The present authors have undertaken an attempt in this book at solving this task. A large number of programs are presented that have enabled us to perform in the process of material presentation both analytical and numeric computations with the aid of a personal computer. All *Mathematica 3.0* programs are stored on the Birkhäuser server. The url address of which is as follows (see Appendix B for further details of our programs):

`http://www.birkhauser.com/book/isbn/0-8176-3995-0`

In addition, we aimed at taking into account the international character of science. Therefore, we considered it necessary to familiarize Western readers in more detail with the achievements of the Russian scientists in the field of fluid mechanics.

The present book has been written on the basis of the lecture courses, which were presented by the authors during the past few years at the Novosibirsk State Technical University and at the Novosibirsk State University. The lectures were intended for graduate and postgraduate students that have already attended the introductory lecture courses in FM. At the same time, this book gives all of the necessary FM concepts and the derivations of all formulas are presented. There are also a large number of problems with the solutions. All of these features enable one to use this book both for an initial and a deeper study of FM.

Note that, although we present FM as a theoretical discipline, its development is closely related to the experiment and the practical needs of industry. The role of experiment in the formation of the basic concepts and closing relationships of FM is very significant. After these concepts have been established, however, the deductive methods of mathematics take on deciding significance.

The book consists of seven chapters. Chapter 1 presents the basic concepts of continua: the Lagrangian and Eulerian description, tensors of strains and stresses, equations of continuity, momentum, and energy. We use throughout the chapter a tensor invariant form, which does not depend on the choice of coordinate system. Therefore, the presentation of the basic FM concepts is preceded by a brief introduction in tensor analysis. The basic definitions of the tensor analysis, which are used in the following, are briefly introduced. From the very beginning, the strains are not assumed to be small; therefore, various tensors of strains and stresses are introduced that are related to the initial and current configurations at the Lagrangian description and in a fixed Eulerian frame.

In Chapter 2, we present the derivation of the differential equations for continuity, energy, and motion. The concept of local thermodynamic equilibrium as well as the the thermodynamics laws enable us to indicate the general form of the closure relationships for the FM governing equations. We formulate the Hamilton–Ostrogradsky variational principle, which enables us, on the one hand, to find the FM motion equations and, on the other hand, to establish a relation between the integral conservation laws for energy and momentum and the isotropicity and homogeneity properties of space and time.

In Chapter 3, the fundamentals of the similarity and dimension theory as well as the mathematical methods for studying the weak discontinuities (the characteristics) and strong discontinuities in fluid mechanics are presented.

Chapter 4 is devoted to the fundamentals of the theory of dynamics of ideal incompressible fluid. For the ideal fluid, we derive the Bernoulli and Thomson integrals and consider the planar and axisymmetric irrotational flows. We also study the general properties of the vortex flows. We further consider in detail the methods for the solution of problems on the ideal fluid flow around planar and axisymmetric bodies.

Chapter 5 deals with the viscous fluid flows. The Navier–Stokes equations are derived, and a number of the solutions of these equations are obtained at small Reynolds numbers. The basics of the Prandtl's boundary layer theory are presented. Some approaches to the description of turbulent fluid flows are considered.

Chapter 6 is devoted to the gas dynamics of ideal and viscous compressible gases. We present the theory of the Laval nozzle, shock waves, and Riemann waves. We also give the solution of a problem on shock wave structure in a viscous gas. The Chaplygin's theory for the transformation of gas dynamics equations to the hodograph plane is presented in sufficient detail.

Chapter 7 is devoted to a new FM branch, the mechanics of multiphase heterogeneous media. This branch of mechanics has appeared during the past 20–30 years, and it now enjoys a period of intense development. One can speak today about the fact that the general approaches have been developed, which are applicable to the description of an arbitrary multiphase medium. Various methods of averaging belong to them, which enable one to go over to an averaged description, as well as the idea of interpenetrating continua, each of which refers to a corresponding phase.

A mathematical model of the gas–particle flow was historically the first model describing multiphase flows. This was related to the practical applications in the field of two-phase gas dynamics of Laval nozzles and the flow of dusty gas around the flying vehicles. The developed methods were then used in the models of bubbly fluid flows, porous materials, and gas mixtures. For this reason, sufficient attention is paid here to the model of gas–particle flow. It should be noted that there are at present in the literature a number of monographs devoted to the mechanics of multiphase media. These monographs are cited in the list of References. Our presentation is nevertheless different from them, since a number of the results presented in our book belongs to the present authors. This refers to the theory of thin discontinuities in the gas–particle mixtures, which carry a finite surface mass; the continual/discrete model, which enables one to model the flows with the intersection of particles trajectories; the theory of caustics in the pseudogas of particles, which gives a limitation for the total number of particles lying on a caustic as well as the condition for its formation; the theory of shock wave interaction with a particle's cloud within the framework of which an explanation is given for the formation of a collective shock wave upstream of the particle's cloud at a volume concentration of particles of the order of several percents.

A triple numbering of formulas is used in the book. The first number indicates the chapter number, the second number is the section number, and the third number is the formula number in the section.

The authors hope that this book will be useful for students of universities and higher technical colleges as well as for specialists working in the field of FM.

The authors express their deep gratitude to professional colleagues whose discussions contributed to the elucidation of many complex questions of FM.