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in Mathematics
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Volume 115

Series Editor
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Springer Science+Business Media, LLC

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Pattern Formation in Continuous and Coupled Systems

A Survey Volume

With 101 Illustrations



Springer

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Mathematics Subject Classifications (1991): 34Cxx, 35Bxx, 58Fxx, 76XX, 92Exx

CIP Data Available.

Printed on acid-free paper.

© 1999 Springer Science+Business Media New York
Originally published by Springer-Verlag New York in 1999
Softcover reprint of the hardcover 1st edition 1999

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Production managed by Allan Abrams; manufacturing supervised by Thomas King.
Camera-ready copy prepared by the IMA.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4612-7192-5 ISBN 978-1-4612-1558-5 (eBook)
DOI 10.1007/978-1-4612-1558-5

SPIN 10729787

FOREWORD

This IMA Volume in Mathematics and its Applications

PATTERN FORMATION IN CONTINUOUS AND COUPLED SYSTEMS

is based on the proceedings of a workshop with the same title, but goes beyond the proceedings by presenting a series of mini-review articles that survey, and provide an introduction to, interesting problems in the field. The workshop was an integral part of the 1997–98 IMA program on “EMERGING APPLICATIONS OF DYNAMICAL SYSTEMS.”

I would like to thank Martin Golubitsky, University of Houston (Mathematics) Dan Luss, University of Houston (Chemical Engineering), and Steven H. Strogatz, Cornell University (Theoretical and Applied Mechanics) for their excellent work as organizers of the meeting and for editing the proceedings.

I also take this opportunity to thank the National Science Foundation (NSF), and the Army Research Office (ARO), whose financial support made the workshop possible.

Willard Miller, Jr., Professor and Director

PREFACE

Pattern formation has been studied intensively for most of this century by both experimentalists and theoreticians, and there have been many workshops and conferences devoted to the subject. In the IMA workshop on *Pattern Formation in Continuous and Coupled Systems* held May 11-15, 1998 we attempted to focus on new directions in the patterns literature. In particular, we stressed systems and phenomena that generate new types of pattern (those that appear in discrete coupled systems, those that appear in systems with global coupling, and those that appear in combustion experiments) and on well-known patterns where there has been significant recent development (for example, spiral waves and superlattice patterns).

The participants at this meeting included, in more or less equal parts, experimentalists and theoreticians. One goal was to continue communication between these groups, and we were pleased by the result. Another goal was to familiarize a larger audience with some of the newer directions in the field, and again the result was very satisfying.

With these goals in mind, we decided to produce a nonstandard workshop proceedings. We did not want to publish a collection of research articles, which could have appeared elsewhere as refereed journal articles, nor did we want to publish a list of abstracts. Instead, we attempted to collect a series of mini-review articles of at most 15 to 20 pages (with extensive bibliographies) that would discuss why certain topics are interesting and merit additional research. The response has been quite heartening and we hope that readers will find these reviews a useful entry into the literature.

The articles that appear here have not been refereed — though they have been read for comprehensibility. We thank the authors for their efforts to produce these articles in a timely fashion. We also thank the IMA staff — in particular Patricia V. Brick and Phong Nguyen — for their expert help in producing this volume and doing so so expeditiously.

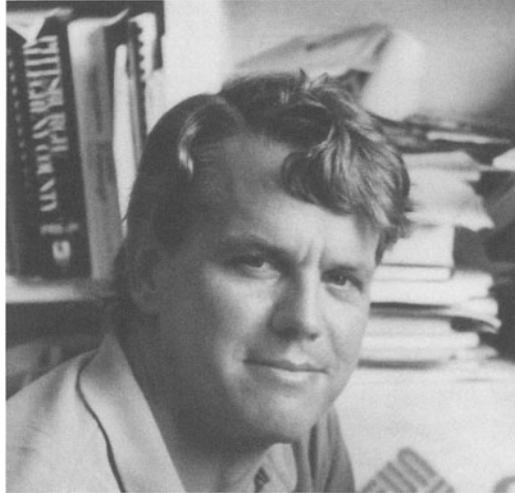
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Dedication

JOHN DAVID CRAWFORD 1954–1998



John David Crawford died on 23 August, 1998 from Burkitt's Lymphoma at the age of 44. This IMA workshop was the last workshop that John David attended — at a time when it was thought that the disease was in remission. John David's presence and active participation added a special dimension to the workshop for those who knew him.

John David graduated with honors from Princeton University in 1977 and with a doctorate in Physics from the University of California at Berkeley in 1983. His thesis on *Hopf Bifurcation and Plasma Instabilities* was written under the direction of Henry Abarbanel. During his career, John David spent six years at the University of California at San Diego, at first in the Physics Department working on non-neutral plasmas and subsequently at the Institute for Nonlinear Science pursuing his interests in bifurcation theory. In 1989 he held visiting positions at the Mathematics Institute at the University of Warwick and at the Institute for Fusion Studies at the University of Texas at Austin. He joined the faculty of the Department of Physics and Astronomy at the University of Pittsburgh in 1990.

John David's interests ranged from the physics of collisionless plasmas to the mathematics of pattern formation. However, there was a common thread: understanding the development and equilibration of instabilities in diverse systems, be they Hamiltonian or dissipative. This workshop focused on pattern formation in continuous systems, a subject to which John David contributed greatly. He worked on developing group-theoretic methods for use in pattern formation studies of dissipative systems and

new techniques for studying bifurcation phenomena in Hamiltonian systems associated with the emergence of an eigenvalue from a continuous spectrum. The former area of research was motivated primarily by his interest in parametrically driven water waves (the Faraday system) and the latter by the beam-plasma instability in the Vlasov-Poisson system.

The Faraday instability is a subharmonic instability and is therefore associated with a Floquet multiplier at -1 . John David's early work with Edgar Knobloch and Hermann Riecke [10]-[12] discussed mode interaction in the Faraday experiment in a circular container, focusing on the dynamics of discrete time- T maps with -1 Floquet multipliers of double multiplicity as appropriate for modes that break the $O(2)$ symmetry of the container. Of particular interest was the classification of the conditions under which the mixed patterns resulting from such interaction drift azimuthally. Such rotating patterns were observed in experiments by Sergio Ciliberto and Jerry Gollub. His subsequent and classic work on the Faraday system in a square container [18] was also motivated by Gollub's experiments. In this work John David focused on understanding the hidden symmetries, both translations and rotations, introduced into the Faraday system by Neumann boundary conditions [18, 21]. These depend on the modes excited and on their degeneracy. John David's observation that as a result there is a significant difference between the Faraday system in a square container and one with D_4 symmetry but in a nonsquare container [17] was confirmed in subsequent experiments by Gollub and David Lane [20]. Related work on parametrically modulated Hopf bifurcation in systems with $O(2)$ symmetry [4, 13] predicted that such modulation would stabilize standing waves even in cases in which traveling waves were preferred in the absence of modulation. This prediction was also confirmed in elegant experiments by Victor Steinberg and David Andereck and their colleagues.

At the same time John David continued his studies of bifurcations in collisionless plasmas. Using the technique of spectral deformation developed in landmark papers with Peter Hislop [8, 9] he was able to understand in detail the appearance of a neutral eigenvalue (or mode) embedded in a continuous spectrum at threshold for instability. In this problem, as in the closely related shear flow problems for ideal fluids, the instability appears when the electron distribution function or shear flow profile are gradually changed, for example, by injecting a beam of fast electrons to create a bump on the tail of the electron distribution or changing the pressure distribution driving the flow. However, because of the presence of the continuum center manifold theory cannot be used to study the resulting bifurcation. John David's understanding of the structure of the linear problem led him to consider the equilibration of the resulting instability using the instability growth rate γ as the bifurcation parameter. In a remarkable paper [23] he showed that in the limit of fixed (i.e. heavy) ions the instability saturates at $O(\gamma^2)$ amplitude, in contrast to the $O(\gamma^{\frac{1}{2}})$ amplitude familiar from dissipative systems. This result is *nonperturbative*, and terms of all orders

contribute to the equilibration as $\gamma \rightarrow 0$ [24]. Thus not only do these instabilities saturate at a much smaller amplitude but they do not have to approach the equilibrium monotonically. The predicted γ^2 “trapping” scaling agrees with numerical and experimental observations. Subsequent work by John David’s student Anand Jayaraman [27, 30] generalized these conclusions to mobile ions showing that the scaling changes to $\gamma^{\frac{5}{2}}$.

While engaged in this work John David realized that similar mathematics applies to the Kuramoto model of phase-coupled oscillators. This model consists of many globally coupled oscillators with frequencies drawn from a prescribed frequency distribution and exhibits a remarkable “phase transition” as the strength K of the interaction increases in which the oscillators begin to phase-lock. As in the Vlasov-Poisson system the stability problem for the incoherent state has a continuous spectrum and this state loses stability at $K = K_c$ when an unstable eigenvalue pops out of the neutral continuum. As a result of a calculation to all orders similar to the plasma one, John David showed [22] that the saturated amplitude (the fraction of synchronized oscillators) scales like $(K - K_c)^{\frac{1}{2}}$ for the Kuramoto model but scales like $K - K_c$ for more general couplings than assumed by Kuramoto [25, 29]. These results resolve analytically several long-standing issues in both theoretical and numerical studies of this important model.

John David wrote two influential review articles, one on basic bifurcation theory [19] and one with Edgar Knobloch on the use of equivariant bifurcation theory for studies of pattern formation in fluid dynamics [16]. A bibliography of John David’s contributions to pattern formation and bifurcation theory is included below.

John David was a consummate scholar, devoted to deep understanding of important and challenging problems. His solutions to these problems were always innovative offering a fresh perspective. At home both in physics and mathematics John David was an invaluable colleague, generous with his time and ideas, and a rare knack for explaining scientific principles to friends, colleagues and students. His lectures were a model of clarity and he was a much sought-after speaker. At the workshop his delight in being back in the milieu he so loved was almost palpable. He will be greatly missed by all of us.

Edgar Knobloch
October, 1998

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