

# Computer Networks and Systems

## Queueing Theory and Performance Evaluation

Third Edition

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Thomas G. Robertazzi

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Queueing Theory and  
Performance Evaluation

Third Edition

With 116 Figures



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*To Marsha, Rachel, and Deanna*

# Preface

Statistical performance evaluation has assumed an increasing amount of importance as we seek to design more and more sophisticated communication and information processing systems. The ability to predict a proposed system's performance before one constructs it is an extremely cost effective design tool.

This book is meant to be a first-year graduate level introduction to the field of statistical performance evaluation. It is intended for people who work with statistical performance evaluation including engineers, computer scientists and applied mathematicians. As such, it covers continuous time queueing theory (chapters 1-4), stochastic Petri networks (chapter 5), discrete time queueing theory (chapter 6) and recent network traffic modeling work (chapter 7). There is a short appendix at the end of the book that reviews basic probability theory. This material can be taught as a complete semester long course in performance evaluation or queueing theory. Alternatively, one may teach only chapters 2 and 6 in the first half of an introductory computer networking course, as is done at Stony Brook. The second half of the course could use a more protocol oriented text such as ones by Saadawi [SAAD] or Stallings [STAL].

What is new in the third edition of this book? In addition to the well received material of the second edition, this edition has three major new features. First of all, solutions to all of the chapter 2 through 6 problems are being published in a separate volume by Springer-Verlag. I believe this will be a great aid to students and engineers, computer scientists and applied mathematicians seeking to learn this subject. The second feature is a new chapter 7 on network traffic models that have been the subject of much interest since the publication of the second edition. Chapter 7 includes discussions of continuous and discrete time models, burstiness, self-similar traffic modeling and solution techniques. Lastly, I have added sixteen new problems to chapter 6 on discrete time queueing systems. Many of these involve switching.

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My appreciation of this material has been enhanced by interaction with my students in Stony Brook's computer networks and performance evaluation courses. I am grateful to S. Rappaport for encouraging me to teach the performance evaluation course. Thanks are due for editorial assistance for this edition to Springer-Verlag's T. von Foerster and J. Mallozzi and to J. Wolkowicki for the production of this book and the solutions volume. Thanks are due to M. Gerla for reviewing the manuscript of the first edition. Thanks are also due to H. Badr, M.

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Finally, I would like to dedicate this book to my wife, Marsha and my two daughters, Rachel and Deanna, who made writing this book worthwhile.

*T.G.R.*

*Stony Brook, N.Y.*

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