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# Inviscid Fluid Flows

With 45 Illustrations



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# Preface

Applied Mathematics is the art of constructing mathematical models of observed phenomena so that both qualitative and quantitative results can be predicted by the use of analytical and numerical methods. Theoretical Mechanics is concerned with the study of those phenomena which can be observed in everyday life in the physical world around us. It is often characterised by the macroscopic approach which allows the concept of an element or particle of material, small compared to the dimensions of the phenomena being modelled, yet large compared to the molecular size of the material. Then atomic and molecular phenomena appear only as quantities averaged over many molecules. It is therefore natural that the mathematical models derived are in terms of functions which are continuous and well behaved, and that the analytical and numerical methods required for their development are strongly dependent on the theory of partial and ordinary differential equations. Much pure research in Mathematics has been stimulated by the need to develop models of real situations, and experimental observations have often led to important conjectures and theorems in Analysis. It is therefore important to present a careful account of both the physical or experimental observations and the mathematical analysis used.

The authors believe that Fluid Mechanics offers a rich field for illustrating the art of mathematical modelling, the power of mathematical analysis and the stimulus of applications to readily observed phenomena. Mathematical models in Fluid Mechanics are frequently nonlinear and their solution is challenging. The material selected for discussion in these notes has been chosen both for its mathematical interest and its physical relevance. It is hoped that the topics chosen provide instructive examples

of a mathematical investigation of a real problem and that, by analogy, the reader may be equipped to tackle an apparently unrelated problem in a different field. We have restricted the discussion in these notes to the flow of *inviscid* (not viscous) fluids. Even with this restriction it is possible to describe striking phenomena such as tidal waves and sonic booms.

For the past ten years, the notes have been used at Oxford University in association with a course given to final year undergraduates and first year graduates in Mathematics. It is assumed that the reader has covered standard elementary material on inviscid incompressible hydrodynamics and has had an introduction to partial differential equations and wave motion. Sections which are asterisked are included for completeness but may be omitted if a shorter course is required. Most students taking the Oxford course would also in the same year take a corresponding course on viscous flow in which the ideas of boundary layer theory would be discussed in detail, together with the relevance of the inviscid model. A suitable reference for this material is Batchelor [1]. Exercises are given at the end of each chapter, many of which are modified versions of examination questions in the Oxford Final Honour School of Mathematics.

We are both very grateful for the valuable advice and criticism given by Dr. J. R. Ockendon in the preparation of these notes.

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