Applied Mathematical Sciences | Volume 52

Applied Mathematical Sciences

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Springer Science+Business Media, LLC

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Variational Inequalities and Flow in Porous Media

Springer Science+Business Media, LLC

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AMS Classification: 76S05, 49A29

Library of Congress Cataloging in Publication Data
Chipot, M. (Michel)
Variational inequalities and flow in porous media.
(Applied mathematical sciences ; v. 52)
Bibliography: p.
Includes index.
1. Fluid dynamics. 2. Porous materials.
3. Variational inequalities (Mathematics) I. Title.
II. Series: Applied mathematical sciences
(Springer-Verlag New York Inc.) ; v. 52.
QA1.A647 vol. 52 [QA911] 510s [532'.051] 84-5598

With 13 Illustrations

9 8 7 6 5 4 3 2 1

ISBN 978-0-387-96002-9 ISBN 978-1-4612-1120-4 (eBook)
DOI 10.1007/978-1-4612-1120-4

Preface

These notes are the contents of a one semester graduate course which I taught at Brown University during the academic year 1981-1982. They are mainly concerned with regularity theory for obstacle problems, and with the dam problem, which, in the rectangular case, is one of the most interesting applications of Variational Inequalities with an obstacle.

Very little background is needed to read these notes. The main results of functional analysis which are used here are recalled in the text. The goal of the two first chapters is to introduce the notion of Variational Inequality and give some applications from physical mathematics. The third chapter is concerned with a regularity theory for the obstacle problems. These problems have now invaded a large domain of applied mathematics including optimal control theory and mechanics, and a collection of regularity results available seems to be timely. Roughly speaking, for elliptic variational inequalities of second order we prove that the solution has as much regularity as the obstacle(s). We combine here the theory for one or two obstacles in a unified way, and one of our hopes is that the reader will enjoy the wide diversity of techniques used in this approach.

The fourth chapter is concerned with the dam problem. This problem has been intensively studied during the past decade (see the books of Baiocchi-Capelo and Kinderlehrer-Stampacchia in the references). The relationship with Variational Inequalities has already been quoted above. Starting with a new point of view introduced by Brezis-Kinderlehrer-Stampacchia, and in a different setting by Alt, we develop the theory for general domains, and with relatively elementary techniques we give the main results on the subject including a first study of the free boundary (i.e., the one limiting the wet set of the porous medium considered).

I am especially indebted to H. Brezis from whom I learned most of this subject.

I wish also to express my thanks to my colleagues in the Applied Mathematics Division of Brown University and particularly to Constantine Dafermos and Jack Hale who helped to create such a friendly atmosphere in the Division.

Thomas Sideris and Jalal Shatah helped me to correct the manuscript; I thank them very much for their friendly assistance.

Finally, I thank Roberta Weller and Katherine MacDougall for the excellent typing of the manuscript.

Michel Chipot
Providence, R.I.
February 1982

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