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K. Yosida

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A Theory of Hyperfunctions

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K. Yosida
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Preface

In the end of the last century, Oliver Heaviside inaugurated an *operational calculus* in connection with his researches in electromagnetic theory. In his operational calculus, the *operator of differentiation* was denoted by the symbol "p". The explanation of this operator p as given by him was difficult to understand and to use, and the range of the validity of his calculus remains unclear still now, although it was widely noticed that his calculus gives correct results in general.

In the 1930s, Gustav Doetsch and many other mathematicians began to strive for the mathematical foundation of Heaviside's operational calculus by virtue of the *Laplace transform*

$$\int_0^\infty e^{-pt} f(t)dt.$$

However, the use of such integrals naturally confronts restrictions concerning the growth behavior of the numerical function $f(t)$ as $t \to \infty$.

At about the midcentury, Jan Mikusiński invented the *theory of convolution quotients,* based upon the *Titchmarsh convolution theorem:*

> If $f(t)$ and $g(t)$ are continuous functions defined on $[0,\infty)$ such that the *convolution* $\int_0^t f(t-u)g(u)du \equiv 0$, then either $f(t) \equiv 0$ or $g(t) \equiv 0$ must hold.

The convolution quotients include the *operator of differentiation* "s" and related operators. Mikusiński's operational calculus gives a satisfactory basis of Heaviside's operational calculus; it can be applied successfully to *linear ordinary differential equations with constant coefficients* as well as to the *telegraph equation* which includes both the *wave and heat equations with constant coefficients*.

The aim of the present book is to give a simplified exposition as well as an extension of Mikusiński's operational calculus.

As for the *simplification*, I should like to mention two points 1^o and 2^o below.

1^o. We give a plain proof of the Titchmarsh convolution theorem by making use of the well-known Liouville Theorem in *analytic function theory*.

2^o. For solving linear ordinary differential equations with constant coefficients, we need not rely upon the Titchmarsh convolution theorem. We need only a rather trivial theorem:

Let $f(t)$ be continuous for $0 \leqq t < \infty$ and $\int_0^t f(u)du \equiv 0$.
Then $f(t) \equiv 0$.

As for the *extension*, I should like to mention the following point 3^o.

3^o. We define the *general power* $(s-\alpha)^\gamma$ of the operator $(s-\alpha)$ (α and γ are complex numbers), by making use of the *general binomial theorem* in *analytic function theory:*

$$(1-z)^\gamma = \sum_{k=0}^{\infty} \binom{\gamma}{k}(-z)^k \quad \text{(convergent for } |z| < 1).$$

Then, by virtue of the general power $(s-\alpha)^\gamma$, we can *solve algebraically* the so-called *Laplace's differential equation:*

$$(a_2t+b_2)y''(t) + (a_1t+b_1)y'(t) + (a_0t+b_0)y(t) = 0,$$

where both the a's and b's are complex numbers and $a_2 \neq 0$. This equation includes Bessel's, Laguerre's and confluent hypergeometric differential equations and the like. It is to be noted here that Henri Poincaré and Emile Picard inaugurated the treatment of such differential equations by the Laplace transform combined with subtle contour integrations in the complex plane.

The present book is a revised and enlarged (by 1^o and 3^o) English Edition of the author's book Operational Calculus, written in Japanese and published by the University of Tokyo Press (1982). The English translation was done by the author.

The author gratefully acknowledges fine help from many friends; Hiroshi Fujita, Heinz Götze, Shûichi Okamoto, Shigetake Matsuura, Kyûya Masuda and Jan Mikusiński. Fujita kindly invited the author to write the Japanese Edition for the U.T.P. Götze of the Springer-Verlag kindly suggested the English edition. The above mentioned 1^o and 2^o were obtained

by the author jointly with Matsuura and Okamoto, respectively. Masuda was a fine critic on 3° during its maturity. Mikusiński's fine work aroused the author's interest towards the operational calculus. To them all, including the U.T.P. and the Springer-Verlag, Inc., I express my sincere thanks.

Kôsaku Yosida

Kamakura
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