Part IV
Inverse Stability, Consistency and Convergence for Initial Value Problems in Partial Differential Equations

In this part, we first establish in a general framework a convergence theory for numerical methods approximating initial value problems. This analysis is made possible by incorporating the given problem and its approximations into the setting of Chapter 6. First of all, however, we have to specify the discrete approximations and discrete convergences underlying the specific problem area considered. They have already been provided, together with the verification of the corresponding properties, by the treatment of corresponding examples in Section 5.3 of Part II. We shall often refer to Chapter 4, in which a series of examples of initial value problems and appropriate numerical methods have been introduced and, moreover, represented in suitable operator notation.

The underlying concepts are again inverse stability and consistency which ensure discrete convergence in a sense appropriate for the present special framework. We shall in Chapter 11 develop the corresponding convergence theory. According to our investigations in Part II, a basic requirement is the equicontinuous equidifferentiability of the approximating mappings. This will be verified at the end of Chapter 11 for several classes of examples. It is worth noticing that here - as well as later in Chapter 12 - the choice of norms in the underlying spaces is of paramount importance.

All of Chapter 12 is dedicated to a study of inverse stability. We establish and apply several special criteria for inverse stability. Positivity properties, for example, guarantee inverse stability with respect to the supremum norm (in spaces of grid functions), Fourier methods provide a tool for proving inverse stability with respect to discrete L^2-

norms. With respect to such norms, the von Neumann stability criterion, basic for the classical Lax-Richtmyer theory, will at times follow from our general concept of inverse stability in a very special situation. The above mentioned special criteria, along with appropriate techniques for verifying them, again rely strongly on the choice of underlying norms.

Applying the general convergence results from Chapter 11, along with the stability criteria of Chapter 12, we are finally able to carry out in Chapter 13 a convergence analysis of special methods by investigating the behavior of the associated truncation errors.