Part III
Convergence Analysis for Approximate Solutions to Boundary-Value Problems and Integral Equations

In this part of the book, we analyze the stability, consistency, and convergence properties of approximation methods for boundary-value problems and integral equations by applying the convergence theory from Part II.

On the one hand, the general concepts in our convergence theory will be further elucidated by reformulating the examples already discussed in Part II in a more concrete setting. On the other hand, we demonstrate the efficiency of our theory by obtaining results, including two-sided error estimates, for the most diverse classes of problems and approximation methods.

Compactness arguments are applicable to the theoretical analysis of approximation methods for both boundary-value problems and integral equations, and account for treating these two different problem areas in this portion of the text. The methods for approximating boundary-value problems are further subdivided into finite-difference methods and projection methods (for the associated variational formulation). Besides compactness properties, maximum principles will be used to analyze finite-difference methods. An analysis of projection methods does not require compactness arguments but techniques are applied which we summarize as variational principles.

The applications point out that a powerful convergence theory in a suitable general framework constitutes an essential basis for a successful analysis of specific problems, but that a hard analysis is often required to verify stability, inverse stability, discrete compactness, and consistency. Thus, we present only a limited number of examples to show the applicability of our theory as well as to provide analytical techniques.

To understand the analysis in each of the chapters of Part III, the reader must have some knowledge of the material presented in the chapters of Part I according to the following diagram:

Part I	Part III	Problem
Chapter: 1	8	Boundary-value problems
2	9	Variational equations
3	10	Integral equations

The theoretical basis for our analysis, however, is the convergence theory from Part II. Note that, in Sections 5.3 and 5.4, we have provided the discrete approximations underlying the analysis of Chapters 8 and 10 together with important properties of sequences of measures needed for the integral equations.