Part I
Presentation of Numerical Methods

In this preliminary part of the book, we introduce several problem areas and numerous suitable approximation methods. The problems we consider are

boundary-value problems in ordinary and partial differential equations;
integral equations of the second kind; and
initial value problems in partial differential equations.

For the first class of problems, we discuss two different techniques for deriving approximation methods, namely

finite-difference methods, and
projection methods for the associated variational formulation.

The equivalent variational formulation of boundary-value problems, rather than the classical differential equation formulation, serves as a basis for deriving approximations based on projection methods which include well-known Galerkin methods.

The integral equations we consider are approximated via

quadrature methods, and
projection methods.

For the third problem area, initial value problems in partial differential equations, the approximation methods may again be subdivided into two categories,

finite-difference methods, and
Galerkin methods.

The latter are further subdivided into continuous-time and discrete-time Galerkin methods. The similar nomenclature used for the approximation methods of both initial value problems and boundary-value problems indicate that techniques for deriving approximation schemes for the latter play a substantial role in obtaining approximation methods for the former. We consider linear and nonlinear examples in all problem areas and, correspondingly, derive linear and nonlinear methods.

We shall not be able, in general, to give an in-depth presentation of all the various numerical schemes appropriate for the problems we consider. Only at a few places in the text will it be feasible to consider the numerous variants of each respective method. Elsewhere in the text, however, we shall refer the interested reader to the cited literature for a thorough treatment of other methods related to those in the text.

An important goal of Part I is to acquaint the reader with well-known schemes for approximating problems arising in each of the three problem areas. Also, we formulate both the exact and approximate problems as operator equations which are basic for analyzing the stability and convergence properties in Parts III and IV of the text by the general convergence theory developed in Part II. Much of the discussion and formulas presented here will be referred to elsewhere in the text. On these grounds, we would strongly advise the reader to be familiar with the material in Part I.

The numerical methods we develop in this part will moreover be expressed in linear and nonlinear systems of equations which are suitable for numerical calculations; in addition, we shall cite results concerning their solvability.