Universitext

Springer Science+Business Media, LLC

Universitext

Editors (North America): S. Axler, F.W. Gehring, and K.A. Ribet

Aksoy/Khamsi: Nonstandard Methods in Fixed Point Theory
Andersson: Topics in Complex Analysis
Aupetit: A Primer on Spectral Theory
Booss/Bleecker: Topology and Analysis
Borkar: Probability Theory: An Advanced Course
Carleson/Gamelin: Complex Dynamics
Cecil: Lie Sphere Geometry: With Applications to Submanifolds
Chae: Lebesgue Integration (2nd ed.)
Charlap: Bieberbach Groups and Flat Manifolds
Chern: Complex Manifolds Without Potential Theory
Cohn: A Classical Invitation to Algebraic Numbers and Class Fields
Curtis: Abstract Linear Algebra
Curtis: Matrix Groups
DiBenedetto: Degenerate Parabolic Equations
Dimca: Singularities and Topology of Hypersurfaces
Edwards: A Formal Background to Mathematics I a/b
Edwards: A Formal Background to Mathematics II a/b
Foulds: Graph Theory Applications
Friedman: Algebraic Surfaces and Holomorphic Vector Bundles
Fuhrmann: A Polynomial Approach to Linear Algebra
Gardiner: A First Course in Group Theory
Gårding/Tambour: Algebra for Computer Science
Goldblatt: Orthogonality and Spacetime Geometry
Gustafson/Rao: Numerical Range: The Field of Values of Linear Operators and Matrices
Hahn: Quadratic Algebras, Clifford Algebras, and Arithmetic Witt Groups
Holmgren: A First Course in Discrete Dynamical Systems
Howe/Tan: Non-Abelian Harmonic Analysis: Applications of $SL(2, \mathbb{R})$
Howes: Modern Analysis and Topology
Humi/Miller: Second Course in Ordinary Differential Equations
Hurwitz/Kritikos: Lectures on Number Theory
Jennings: Modern Geometry with Applications
Jones/Morris/Pearson: Abstract Algebra and Famous Impossibilities
Kannan/Krueger: Advanced Analysis
Kelly/Matthews: The Non-Euclidean Hyperbolic Plane
Kostrikin: Introduction to Algebra
Luecking/Rubel: Complex Analysis: A Functional Analysis Approach
MacLane/Moerdijk: Sheaves in Geometry and Logic
Marcus: Number Fields
McCarthy: Introduction to Arithmetical Functions
Meyer: Essential Mathematics for Applied Fields
Mines/Richman/Ruitenburg: A Course in Constructive Algebra
Moise: Introductory Problems Course in Analysis and Topology
Morris: Introduction to Game Theory
Polster: A Geometrical Picture Book
Porter/Woods: Extensions and Absolutes of Hausdorff Spaces
Ramsay/Richtmyer: Introduction to Hyperbolic Geometry
Reisel: Elementary Theory of Metric Spaces
Rickart: Natural Function Algebras

(continued after index)

L.R. Foulds

Graph Theory Applications

With 90 Illustrations

Springer

L. R. Foulds
Department of Management Systems
University of Waikato
Hamilton, New Zealand

Mathematics Subject Classification (1991): 05-01, 49-10, 90-01

Section 7.6 is based on material from Chapter 4 of *Digraphs: Theory and Techniques*, by Robinson and Foulds and published by Gordon and Breach in 1980. This material is included here with the permission of Gordon and Breach.

Library of Congress Cataloging-in-Publication Data
Foulds, L.R., 1948–
Graph theory applications / L.R. Foulds.
p. cm. — (Universitext)
Includes bibliographical references and index
ISBN 978-0-387-97599-3 ISBN 978-1-4612-0933-1 (eBook)
DOI 10.1007/978-1-4612-0933-1
1. Graph theory. I. Title. II. Series.
QA166.F68 1991
511´.5—dc20 91-20590

Printed on acid-free paper.

Photocomposed copy prepared using the author's TEX file.

9 8 7 6 5 4 3

ISBN 978-0-387-97599-3

Preface

Over the last 30 years graph theory has evolved into an important mathematical tool in the solution of a wide variety of problems in many areas of society. The purpose of this book is to present selected topics from this theory that have been found useful and to point out various applications. Some important theoretical topics have been omitted as they are not essential for the applications in Part II. Hence Part I should not be seen as a well-rounded treatise on the theory of graphs. Some effort has been made to present new applications that do not use merely the notation and terminology of graphs but do actually implement some mathematical results from graph theory. It has been written for final undergraduate year or first year graduate students in engineering, mathematics, computer science, and operations research, as well as researchers and practitioners with an interest in graph theoretic modelling. Suggested plans for the reading of the book by people with these interests are given later. The book comprises two parts. The first is a brief introduction to the mathematical theory of graphs. The second is a discussion on the applications of this material to some areas in the subjects previously mentioned. It is, of course, possible to read only the first part to attempt to gain an appreciation of the mathematical aspects of graph theory. However even the purest of mathematicians is strongly recommended to delve seriously into the second part. This is because the theory of graphs and the applications of graphs are inextricably intertwined. Much of the mathematical theory of graphs has arisen out of attempts to solve practical problems. So to ignore the utility of graph theory is to ignore a major part of its importance.

The text evolved out of the experience of the author in teaching the material to students in mathematics at Massey University, operations research at the University of Canterbury, operations management at the University of Waikato, (all in New Zealand) engineering at the University of Florida,

and management information systems at University College Dublin. It contains exercises which the reader is urged to try. As with all disciplines, you cannot master graph theory without getting your hands dirty. Graph theory is not a spectator sport!

The first 10 chapters, making up the first part, are organised as follows. We begin with an introductory chapter which introduces a little historical background and the fundamental notions. It is assumed in all later chapters that the reader is familiar with this material. Chapter 2 is concerned with connectivity — a concept which is basic for many of the later chapters. One of the most important classes of graphs is that of trees and this is dealt with in Chapter 3. The material in the next chapter, on traversability, is of theoretical interest in its own right but is also of practical importance in operations research, covered in Chapter 12. Chapter 5, on planarity, illustrates that the topic is more than just topology. It is necessary for the sections on layout in Chapter 14. Chapter 6, on the matrices of a graph, is essential for a later discussion, on graph theoretic algorithms. Chapter 7 is a necessarily brief account of directed graphs, called digraphs, and their important special case; the network. Chapter 8, on covering, dominance, and matching has application in industrial engineering and other disciplines. Chapter 9 covers graph theoretic algorithms. In Chapter 10 we make a brief excursion into the world of matroids, where there are applications in electrical engineering, among other areas. Part II has mainly longer chapters explaining the application of the above-mentioned material in various branches of engineering, operations research, and science. No attempt has been made to make this part encyclopaedic. Rather, due to limititations of space and for other reasons, just a few applications have been presented in some depth. They are intended to give some impression of the power and wide utility of graph theory.

Part I is suitable as a one-semester course in mathematics or engineering and this could be followed by a second semester covering the applications in Part II. Other one-semester sequences are given later.

A few of the chapter sections and exercises are starred. These require a greater level of mathematical maturity. They may be skipped without loss of continuity.

The author would like to thank the University of Waikato, and University College Dublin. The former institution allowed the author to write this book in the course of his employment. The latter institution hosted the author while he was on sabbatical leave during the 1988–89 year, (along with Trinity College, Dublin) during which time the book was tested on its students.

The author is also grateful to Nikki Sayer who typed the entire manuscript, to Erica Harris for wordprocessing the manuscript using TEX, to Frank Bailey and Doreen Whitehead of the Draughting Department at the University of Waikato for drawing the figures, to Derek O'Connor and Fergus Gaines of University College Dublin, Takeo Yamada and his students at the National Defense Academy, Yokosuka, Japan, Horst Hamacher of the University of Kaiserslautern, Germany, and to Professor J. Labelle of the University of Montreal, who all read the entire manuscript and who suggested many valuable improvements, and to the staff of Springer-Verlag for their able cooperation.

L.R. Foulds
Hamilton, New Zealand

To Jacqueline Beaton
The LFJB

Contents

Teaching Plans

The book is suitable as a course in a number of disciplines. Here are some chapter sequences:

Mathematics:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.

Computer Science:

1, 2, 3, 4, 5, 6, 7, 8, 9, 11.

Management Science or Operations Research:

1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12.

Industrial or Systems Engineering:

1, 2, 3, 4, 5, 6, 7, 8, 9, 14.

Urban Planning or Civil Engineering:

1, 2, 3, 4, 5, 6, 7, 8, 9, 16.

Electrical Engineering:

1, 2, 3, 4, 5, 6, 7, 8, 9, 13.

Science:

1, 2, 3, 4, 5, 6, 7, 8, 9, 15.

Graph Theory Applications