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(continued after index)

L.R. Foulds

Graph Theory Applications

With 90 Illustrations



Springer

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Preface

Over the last 30 years graph theory has evolved into an important mathematical tool in the solution of a wide variety of problems in many areas of society. The purpose of this book is to present selected topics from this theory that have been found useful and to point out various applications. Some important theoretical topics have been omitted as they are not essential for the applications in Part II. Hence Part I should not be seen as a well-rounded treatise on the theory of graphs. Some effort has been made to present new applications that do not use merely the notation and terminology of graphs but do actually implement some mathematical results from graph theory. It has been written for final undergraduate year or first year graduate students in engineering, mathematics, computer science, and operations research, as well as researchers and practitioners with an interest in graph theoretic modelling. Suggested plans for the reading of the book by people with these interests are given later. The book comprises two parts. The first is a brief introduction to the mathematical theory of graphs. The second is a discussion on the applications of this material to some areas in the subjects previously mentioned. It is, of course, possible to read only the first part to attempt to gain an appreciation of the mathematical aspects of graph theory. However even the purest of mathematicians is strongly recommended to delve seriously into the second part. This is because the theory of graphs and the applications of graphs are inextricably intertwined. Much of the mathematical theory of graphs has arisen out of attempts to solve practical problems. So to ignore the utility of graph theory is to ignore a major part of its importance.

The text evolved out of the experience of the author in teaching the material to students in mathematics at Massey University, operations research at the University of Canterbury, operations management at the University of Waikato, (all in New Zealand) engineering at the University of Florida,

and management information systems at University College Dublin. It contains exercises which the reader is urged to try. As with all disciplines, you cannot master graph theory without getting your hands dirty. Graph theory is not a spectator sport!

The first 10 chapters, making up the first part, are organised as follows. We begin with an introductory chapter which introduces a little historical background and the fundamental notions. It is assumed in all later chapters that the reader is familiar with this material. Chapter 2 is concerned with connectivity — a concept which is basic for many of the later chapters. One of the most important classes of graphs is that of trees and this is dealt with in Chapter 3. The material in the next chapter, on traversability, is of theoretical interest in its own right but is also of practical importance in operations research, covered in Chapter 12. Chapter 5, on planarity, illustrates that the topic is more than just topology. It is necessary for the sections on layout in Chapter 14. Chapter 6, on the matrices of a graph, is essential for a later discussion, on graph theoretic algorithms. Chapter 7 is a necessarily brief account of directed graphs, called digraphs, and their important special case; the network. Chapter 8, on covering, dominance, and matching has application in industrial engineering and other disciplines. Chapter 9 covers graph theoretic algorithms. In Chapter 10 we make a brief excursion into the world of matroids, where there are applications in electrical engineering, among other areas. Part II has mainly longer chapters explaining the application of the above-mentioned material in various branches of engineering, operations research, and science. No attempt has been made to make this part encyclopaedic. Rather, due to limitations of space and for other reasons, just a few applications have been presented in some depth. They are intended to give some impression of the power and wide utility of graph theory.

Part I is suitable as a one-semester course in mathematics or engineering and this could be followed by a second semester covering the applications in Part II. Other one-semester sequences are given later.

A few of the chapter sections and exercises are starred. These require a greater level of mathematical maturity. They may be skipped without loss of continuity.

The author would like to thank the University of Waikato, and University College Dublin. The former institution allowed the author to write this book in the course of his employment. The latter institution hosted the author while he was on sabbatical leave during the 1988–89 year, (along with Trinity College, Dublin) during which time the book was tested on its students.

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To Jacqueline Beaton
The LFJB

Contents

Preface	v
Teaching Plans	xv
Part I: The Theory Of Graphs	1
Chapter 1: BASIC IDEAS	3
History	3
Initial Concepts	9
Summary	15
Exercises	15
Chapter 2: CONNECTIVITY	17
Introduction	17
Elementary Results	19
Structure Based on Connectivity	22
Summary	24
Exercises	25

Chapter 3:	TREES	27
	Characterizations	27
	Theorems on Trees	30
	Tree Distances	30
	Binary Trees	32
	Tree Enumeration	35
	Spanning Trees	37
	Fundamental Cycles	38
	Summary	39
	Exercises	41
Chapter 4:	TRAVERSABILITY	43
	Introduction	43
	Eulerian Graphs	43
	Hamiltonian Graphs	46
	Summary	50
	Exercises	51
Chapter 5:	PLANARITY	53
	The Utilities Problem	53
	Plane and Planar Graphs	55
	Planar Graph Representation	55
	Planarity Detection	60
	Duality	64
	Thickness and Crossing Numbers	70
	Summary	73
	Exercises	73
Chapter 6:	MATRICES	75
	The Adjacency Matrix	76
	The Incidence Matrix	77
	The Cycle Matrix	80
	The Cut-Set Matrix	84
	The Path Matrix	90
	Summary	91
	Exercises	91

Chapter 7:	DIGRAPHS	93
	Connectivity	93
	Traversability	98
	Directed Trees	100
	More Digraph Matrices	100
	The Principle of Directional Duality	107
	Tournaments	108
	Summary	120
	Exercises	120
Chapter 8:	COVERINGS AND COLOURINGS	123
	Covering, Independence, and Domination	124
	Colouring	132
	Matching	134
	Summary	142
	Exercises	143
Chapter 9:	ALGORITHMS	145
	Algorithms	146
	Input	146
	Complexity	149
	Output	160
	Graph Analysis Algorithms	161
	Graph Optimization Algorithms	174
	Summary	180
	Exercises	180
Chapter 10:	MATROIDS	183
	Introduction	184
	Duality	186
	The Greedy Algorithm	188
	Summary	191
	Exercises	191

Part II: Applications	193
Chapter 11: MISCELLANEOUS APPLICATIONS	195
Social Sciences	197
Economics	199
Geography	202
Architecture	207
Puzzles and Games	210
Summary	220
Exercises	221
Chapter 12: OPERATIONS RESEARCH	225
Operations Research and Graph Theory	226
Graph Theoretic Algorithms in OR	226
Graph Theoretic Heuristics in OR	231
Digraphs in OR	233
Optimization Algorithms	234
Transportation Networks: Advanced Models	261
Summary	265
Exercises	265
Chapter 13: ELECTRICAL ENGINEERING	269
Electrical Network Analysis	269
Printed Circuit Design	279
Summary	289
Exercises	289
Chapter 14: INDUSTRIAL ENGINEERING	291
Production Planning and Control	291
Facilities Layout	292
Summary	318
Exercises	319

Chapter 15:	SCIENCE	323
	Physics	323
	Chemistry	324
	Biology	328
	Summary	340
	Exercises	340
Chapter 16:	CIVIL ENGINEERING	343
	Earthwork projects	343
	Traffic Network Design	344
	Summary	358
	Exercises	358
	Further Reading	361
	Bibliography	365
	Index	379

Part II: Applications	193
Chapter 11: MISCELLANEOUS APPLICATIONS	195
Social Sciences	197
Economics	199
Geography	203
Architecture	207
Puzzles and Games	210
Summary	220
Exercises	221
Chapter 12: OPERATIONS RESEARCH	225
Operations Research and Graph Theory	226
Graph Theoretic Algorithms in OR	226
Graph Theoretic Heuristics in OR	231
Digraphs in OR	233
Optimization Algorithms	234
Transportation Networks: Advanced Models	261
Summary	265
Exercises	265
Chapter 13: ELECTRICAL ENGINEERING	269
Electrical Network Analysis	269
Printed Circuit Design	279
Summary	289
Exercises	289
Chapter 14: INDUSTRIAL ENGINEERING	291
Production Planning and Control	291
Facilities Layout	292
Summary	319
Exercises	320

Teaching Plans

The book is suitable as a course in a number of disciplines. Here are some chapter sequences:

Mathematics:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.

Computer Science:

1, 2, 3, 4, 5, 6, 7, 8, 9, 11.

Management Science or Operations Research:

1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12.

Industrial or Systems Engineering:

1, 2, 3, 4, 5, 6, 7, 8, 9, 14.

Urban Planning or Civil Engineering:

1, 2, 3, 4, 5, 6, 7, 8, 9, 16.

Electrical Engineering:

1, 2, 3, 4, 5, 6, 7, 8, 9, 13.

Science:

1, 2, 3, 4, 5, 6, 7, 8, 9, 15.

Graph Theory Applications