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Anadijiban Das

The Special Theory of Relativity

A Mathematical Exposition

With 27 Illustrations

Springer

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Dedicated to Sri Gadadhar Chattopadhyaya

Preface

The material in this book is presented in a logical sequence rather than a historical sequence. Thus, we feel obligated to sketch briefly the history of the special theory of relativity. The brilliant experiments of Michelson and Morley in 1887 demonstrated the astonishing fact that the speed of light is independent of the state of relative linear motion of the source of light and the observer of the light. This fact necessitates the modification of the usual Galilean transformation (between two relatively moving observers), which tacitly assumes that time and space are absolute.

Fitzgerald in 1889 and Lorentz in 1892 altered the Galilean transformation by introducing a length contraction in the direction of relative motion. This contraction explained the Michelson–Morley experiment, but it was viewed by both Fitzgerald and Lorentz as a mathematical trick only and *not* indicative of the nature of reality. In 1898 Larmor introduced a similar time dilation in an attempt to find the transformations which leave Maxwell's equations invariant. Lorentz also introduced the time dilation independently sometime before 1904. Poincaré in 1905 also discovered the Lorentz transformation and asserted that it was the fundamental invariance group of nature. Einstein in 1905 discovered the Lorentz transformation from physical considerations. Einstein, alone among these mathematical physicists, recognized the philosophical implications of the Lorentz transformation in that it rejected the commonly held notion that space and time were both absolute. He postulated the equivalence of all inertial frames of reference (moving with constant velocities relative to each other) with regard to the formulation of natural laws. Furthermore, he recognized and postulated that the speed of light is the maximum speed of propagation of any physical action. Therefore, the speed of light must be invariant for all inertial observers. Thus the Michelson-Moreley experiment was reconciled with theory. Minkowski, a mathematician, combined both physical postulates of Einstein into one mathematical axiom. This axiom is that "all natural laws must be expressible as tensor field equations on a (flat) absolute space–time manifold." Thus, in that there is no preferred inertial frame for the formulation of natural laws, a universal democracy is postulated to exist among all inertial observers. This axiom is called the Principle of Special Relativity. Many experiments involving atoms and subatomic particles have verified the essential validity of this principle.

In the first chapter we introduce axiomatically the four-dimensional Minkowski vector space. This vector space is endowed with a nondegenerate inner product which is *not* positive definite. Therefore, the concepts of the *norm* (or length) of a four-vector and of the *angle* between two four-vectors have to be *abandoned.* A Lorentz mapping is introduced as an inner product preserving linear mapping of Minkowski vector space into itself.

In Chapter 2 we introduce the flat Minkowski space–time manifold with a proper axiomatic structure. It is proved that the transformation from one Minkowski chart to another must be given by a Poincaré transformation (or an inhomogeneous Lorentz transformation). The *conceptual difference* between a Lorentz transformation of coordinate charts and a Lorentz mapping of the tangent (Minkowski) vector space is clearly displayed. Minkowski tensor fields on the flat space–time are also defined.

In the third chapter, by applications of a particular Lorentz transformation (the "boost"), length contraction, time retardation, and the composition of velocities are explained. The group structure of the set of all Lorentz transformations is demonstrated, and real representations of the Lorentz group are presented. The proper orthochronous subgroup is defined and discussed also.

The fourth chapter defines the spinor space (a two-dimensional complex vector space) and the properties of spinors. Bispinor space (a four-dimensional complex vector space) is also introduced. It is shown that a unimodular mapping of spinor space can induce a proper, orthochronous Lorentz mapping on Minkowski vector space. Furthermore, a unimodular mapping of bispinor space is shown to induce a general Lorentz mapping of Minkowski vector space.

In Chapter 5 prerelativistic mechanics is briefly reviewed. In the setting of prerelativistic mechanics in space and time, $\mathbb{E}_3 \times \mathbb{R}$, the momentum conjugate to the time variable turns out to be the negative of energy! After this, the relativistic mechanics is investigated. The Lagrangian is assumed to be a positive homogeneous function of degree one in the velocity variables (which makes the generalized Hamiltonian identically zero!). Examples from electromagnetic theory and the linearized gravitational theory of Einstein are worked out.

In Chapter 6 the relativistic (classical) field theory is developed. Noether's theorem (essential for the differential conservation laws) is rigorously proved. As examples of special fields, the Klein–Gordon scalar field, the electromagnetic tensor field, nonabelian gauge fields, and the Dirac bispinor field are presented. However, at the present level of treatment, gauge fields are *not* derived as connections in a fibre bundle over the base (Minkowski) manifold. In each chapter, examples and exercises of various degrees of difficulty are provided.

Chapter 7 deals with a research topic, namely, classical fields in the eight-dimensional extended (or covariant) phase space. Historically, Born and Yukawa advocated the extended phase space on the basis of the principle of *reciprocity* (covariance under the canonical transformation $\hat{p} = -q$, $\hat{q} = p$). In recent years, Caianello and others have considered the principle of *maximal proper acceleration* arising out of the extended phase space geometry. We ourselves have done some research on classical fields in the eight-dimensional phase space. We can obtain, in a certain sense, a unified meson field and a unification of fermionic fields. These fields, however, contain *infinitely* many modes or particles.

We have *changed* the usual notation for the Lorentz metric η_{ij} in favor of d_{ij} (since η_{ijkl} is used for the pseudotensor) and $\gamma \equiv (1 - v^2)^{-1/2}$ in favor of $\beta \equiv (1 - v^2)^{-1/2}$ (since γ is used to denote a curve).

This book has grown out of lectures delivered at Jadavpur University (Calcutta), University College of Dublin, Carnegie–Mellon University, and mostly at Simon Fraser University (Canada). The material is intended mainly for students at the fourth and the fifth year university level. We have taken special care to steer a *middle course* between abstruse mathematics and theoretical physics, so that this book can be used for courses in special relativity in both mathematics and physics departments. Furthermore, the material presented here is a suitable prerequisite for further study in either general relativity or relativistic particle theory.

In conclusion, I would like to acknowledge gratefully several people for various reasons. I was fortunate to learn the subject of special relativity from the late Professor S. N. Bose F.R.S. (of Bose–Einstein statistics) in Calcutta University. I also had the privilege for three years of being a research associate of the late Professor J. L. Synge F.R.S. at the Dublin Institute for Advanced Studies. Their influence, direct or indirect, is evident in the presentation of the material (although the errors in the book are solely due to me!). In preparation of the manuscript, I have been helped very much by Dr. Ted Biech, who typed the manuscript and suggested various improvements. Mrs. J. Fabricius typed the difficult Chapter 7. Mrs. E. Carefoot drew the diagrams. Dr. Shounak Das has suggested some literary improvements. I also owe thanks to many of my students for stimulating discussions during lectures.

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