PART TWO

ADVANCED TOPICS

Paris, 1900

AN ADDRESS

"*As long as a branch of science offers an abundance of problems, so long is it alive: a lack of problems foreshadows extinction or the cessation of independent development. Moreover, a mathematical problem should be difficult in order to entice us, yet not completely inaccessible, lest it mock our efforts. The mathematicians of past centuries ... knew the value of difficult problems. I remind you only of the 'problem of the [path] of quickest descent,' proposed by Johann Bernoulli. The calculus of variations owes its origin to this and to similar problems. ... it often happens also that the same special problem finds application in the most unlike branches of mathematical knowledge. So, for example, the problem of the shortest line plays a chief and historically important part in the foundations of geometry, in the theory of lines and surfaces, in mechanics, and in the calculus of variations.... And it seems to me that the numerous and surprising analogies and that apparently preestablished harmony which the mathematician so often perceives in the questions, methods and ideas of the various branches of his science, have their origin in [the] ever-recurring interplay between thought and experience....*"

"*It is an error to believe that rigor in the proof is the enemy of simplicity. The very effort for rigor forces us to discover simpler methods of proof.... But the most striking example of my statement is the calculus of variations. The treatment of the first and second variations of definite integrals required in part extremely complicated calculations, and the process applied by the old mathematicians had not the needful rigor. Weierstrass showed us the way to a new and sure foundation. By the examples of the single and double integral I will show briefly, at the close of my lecture, how this way leads at once to a surprising simplification....*"

"Mathematical science is in my opinion an indivisible whole, an organism whose vitality is conditioned upon the connection of its parts. The organic unity of mathematics is inherent in the nature of this science, for mathematics is the foundation of all exact knowledge of natural phenomena. That it may completely fulfill this high destiny, may the new century bring it gifted prophets and many zealous and enthusiastic disciples!"

DAVID HILBERT
At the Second International Congress of Mathematicians[1]

[1] These are fragments of the celebrated lecture in which Hilbert set forth 22 additional problems which have challenged mathematicians of all disciplines in this century. The translation of the complete text from which they were compiled will be found in Vol. 8 of the Bulletin of the A.M.S. (1902) pp. 437–445, 478, 479.