Undergraduate Texts in Mathematics

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Alan F. Beardon

Limits

A New Approach to Real Analysis

Springer

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Preface

Broadly speaking, analysis is the study of limiting processes such as summing infinite series and differentiating and integrating functions, and in any of these processes there are two issues to consider; first, there is the question of whether or not the limit exists, and second, assuming that it does, there is the problem of finding its numerical value. By convention, analysis is the study of limiting processes in which the issue of existence is raised and tackled in a forthright manner. In fact, the problem of existence overshadows that of finding the value; for example, while it might be important to know that every polynomial of odd degree has a zero (this is a statement of existence), it is not always necessary to know what this zero is (indeed, if it is irrational, we may never know what its true value is).

Despite the fact that this book has much in common with other texts on analysis, its approach to the subject differs widely from any other text known to the author. In other texts, each limiting process is discussed, in detail and at length before the next process. There are several disadvantages in this approach. First, there is the need for a different definition for each concept, even though the student will ultimately realise that these different definitions have much in common. Next, there is the repetition of remarkably similar results and proofs (perhaps in the hope that by the third or fourth time they will seem easier than before). Thirdly, a rigorous development of 'school mathematics' (the exponential and trigonometric functions, for instance—and this must surely be one of the initial aims in teaching analysis) requires a combination of results taken from different topics in analysis, and this means that in the conventional approach all this has to be left until late in the development. Finally, and perhaps most significantly, in the traditional approach many students finish the course feeling sure that all the ideas they have met have a common thread but are unable to give substance to this feeling.

In this text, we shall define what is meant by a limit just once, and *all of the subsequent limiting processes will be seen as special cases of this one definition*. Accordingly, the subject matter attains a unity and coherence that is missing in the traditional approach. As a by-product of this, we can talk of differentiation, infinite series, continuity, and so on, as early as we wish and, in particular, when we are discussing a careful treatment of school mathematics. Lest those who know the subject should be worried about the level of difficulty, let it be said now that the general definition of a limit is no more complicated than the definition of an equivalence relation (which is standard fare in almost all first-year university courses in mathematics).

The plan of the book is as follows. Part I comprises two chapters; these include some preliminary material on sets, and on real and complex numbers. Many readers will be able to omit most of the material in these chapters. Part II starts with the definition of a limit and its basic properties (in Chapter 3). Chapter 4 contains three basic results (the Intermediate Value Theorem, the Mean Value inequality, and the Cauchy Criterion, all of which are proved by bisection arguments. Chapter 5 contains a detailed discussion of infinite series, including a treatment of unordered sums (which fits well into the general notion of a limit). Part II ends with Chapter 6, which contains a rigorous account of the exponential and trigonometric functions up to and including the periodicity of the exponential function of a complex variable. One of the benefits of this approach is the availability of this material at an early stage. Parts I and II would suffice for a shorter course on the main ideas in analysis at this level, and there can hardly be a better motivation for the analysis than 'closing' the gaps left in earlier treatments. Part III comprises the standard material in analysis, and even this progresses smoothly, since much follows easily from the earlier basic ideas.

The ideas in this text have their roots in discussions of the limiting processes (nets and filters) in topology around 1940 or so. These ideas are given here in a form that (in the view of the author) is suitable for teaching a first course in analysis. The material covered is the standard material that would be found in any first 'serious' course in analysis. Examples occur throughout the text, and there are routine exercises at the end of each section.

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