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Further Topics on Discrete-Time Markov Control Processes

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For Marina, Adrián, Gerardo, and Andrés
To Julia and Marine

Preface

This book presents the second part of a two-volume series devoted to a systematic exposition of some recent developments in the theory of discrete-time Markov control processes (MCPs). As in the first part, hereafter referred to as "Volume I" (see Hernández-Lerma and Lasserre [1]), interest is mainly confined to MCPs with *Borel* state and control spaces, and possibly *unbounded* costs. However, an important feature of the present volume is that it is essentially self-contained and can be read independently of Volume I. The reason for this independence is that even though both volumes deal with similar classes of MCPs, the assumptions on the control models are usually different. For instance, Volume I deals only with *nonnegative* cost-per-stage functions, whereas in the present volume we allow cost functions to take positive or negative values, as needed in some applications. Thus, many results in Volume I on, say, discounted or average cost problems are not applicable to the models considered here.

On the other hand, we now consider control models that typically require more restrictive classes of control-constraint sets and/or transition laws. This loss of generality is, of course, deliberate because it allows us to obtain more "precise" results. For example, in a very general context, in §4.2 of Volume I we showed the convergence of the value iteration (VI) procedure for discounted-cost MCPs, whereas now, in a somewhat more restricted setting, we actually get a lot more information on the VI procedure, such as the *rate of convergence* (§8.3), which in turn is used to study "rolling horizon" procedures, as well as the existence of "forecast horizons", and criteria for the elimination of nonoptimal control actions. Similarly, in Chapter 10 and Chapter 11, which deal with average cost problems, we obtain many interesting results that are virtually impossible to hold in a context as general as that of Volume I. In the Introduction of each chapter dealing with problems already studied in Volume I we clearly spell out the difference between the corresponding settings in each volume.

Volume I comprises Chapter 1 to Chapter 6, and the present volume contains Chapter 7 to Chapter 12. Chapter 7 introduces background material on weighted-norm spaces of functions and spaces of measures, and on noncontrolled Markov chains. In particular, it introduces the concept of w-geometric ergodicity of Markov chains, where w is a given "weight" (or bounding) function, and also the Poisson equation associated to a transition kernel and a given "charge". Chapter 8 studies *α-discounted cost* (abbreviated α-DC or simply DC) MCPs. The basic idea is to give conditions under which the dynamic programming (DP) operator is a *contraction* with respect to a suitable weighted norm. This contraction property is used to obtain the α-discount optimality (or DP-equation), and to make a detailed analysis of the VI procedure. In Chapter 9 we turn our attention to the *expected total cost* (ETC) criterion. Conditions are given for the ETC function to be well defined (as an extended-real-valued measurable function) and for the existence of ETC-optimal control policies, among other things. A quite complete analysis of the so-called "transient" case is also included (§9.6). Chapter 10 deals with *undiscounted* cost criteria, from average cost (AC) problems to overtaking optimality, passing through the existence of canonical policies, bias-optimal policies, Flynn's opportunity cost, and other "intermediate" criteria. AC problems are also dealt with in Chapters 11 and 12, but from very different viewpoints. Chapter 11 studies *sample path* optimality and *variance* minimization, essentially using probabilistic methods, while Chapter 12 concerns the *expected* AC problem using the linear programming approach. Chapter 12 includes in particular a procedure to approximate by *finite* linear programs the infinite-dimensional AC-related linear programs.

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March 1999

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