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Rigid Analytic Geometry and Its Applications

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Preface

The authors' initial aim in writing this book was to provide an English language version of the now out-of-print book *Géométrie analytique rigide et applications.* In attempting to simply update certain parts, we were compelled to rethink and refine others. Thus the book grew into a more voluminous, as well as a different, publication. Its main purpose remains, however, to provide an easy introduction to the theory of rigid spaces. There is a large number of exercises, offering specific examples as well as more specialized topics not treated in the main text.

This theory has evolved over the last 20 years. Moreover, the appreciation for rigid spaces by researchers in algebraic geometry and number theory is growing. The introduction of rigid spaces by J. Tate had the purpose of describing degenerations of curves and abelian varieties. This theme has been studied and the theory is extended by many authors, e.g., D. Mumford, V. Drinfel'd, Y. Manin, M. Raynaud, H. Grauert, R. Remmert, R. Kiehl, L. Gerritzen, S. Bosch et al. Newer applications, like the Langlands conjecture for function fields, the solution of Abhyankar's problem and rigid cohomology, provide a fruitful interaction between rigid spaces, number theory and algebraic geometry. Some chapters of this book give an introduction to these more advanced themes. As a consequence, the level of exposition (and of the exercises) in this book varies. We will now describe the contents of the various chapters.

The preliminary first chapter reviews the theory of valued fields and gives a quick introduction to Banach spaces and Banach algebras (seen as tools for the theory of affinoid algebras). Chapter 2 treats function theory of one variable on the projective line over an algebraically closed, complete non-archimedean valued field. The idea is to compare this theory with complex function theory of one variable. Moreover, this chapter should build up some intuition for the more technical Chapters 3 and 4 on affinoid algebras and rigid spaces.

Affinoid algebras (the theory is developed in Chapter 3) have many features in common with algebras of finite type over a field. Usually however, the proofs and statements for affinoid algebras are more complicated than for algebras of finite type over a field. The theory of rigid spaces, developed in Chapter 4, is not unlike the theory of schemes. The extra structure of a Grothendieck topology is a complication w.r.t. schemes. There is a natural comparison between formal schemes and rigid spaces. In fact, one can think of the "generic fibre" of a formal scheme as the associated rigid space. Chapters 2, 3 and 4 are updated and enlarged versions of corresponding chapters in [72].

Chapters 5 and 6 on curves and abelian varieties contain more and newer material, especially on stable reduction and Néron models. The basic example, namely the omnipresent Tate curve, has been treated in great detail. This provides the basis for the explicit construction of the Néron models of abelian varieties which are analytic tori (i.e., have "multiplicative reduction"). An exposition is given of the more involved theory of Raynaud concerning the uniformization and the Néron model of general abelian varieties.

Chapter 5 contains a rigid analytic proof of the stable reduction theorem for algebraic curves. It has as consequence a description of the universal analytic covering of an algebraic curve. In Chapter 6, the uniformization of Jacobian varieties is based on this universal analytic covering. This provides a link between the uniformization of an algebraic curve and Raynaud's uniformization of an abelian variety.

Chapter 7 builds up and tries to unify the various aspects and ideas concerning overconvergence, the theory of points, Monsky–Washnitzer cohomology and rigid cohomology. Again this describes the work of many researchers. Recently, the Monsky–Washnitzer cohomology has found an application in cryptography. Kedlaya's algorithm, which counts points on hyperelliptic curves over a finite field, is discussed in detail. The purpose of Chapter 8 is to give a relatively down-to-earth basis for rigid étale cohomology. For an exposition of the important application to Drinfel'd's work on the Langlands conjecture for function fields we refer to [6], Chapters 11 and 12. We have only touched upon the importance of rigid étale cohomology for the study of p-adic symmetric spaces. The latter theory and the connection with p-adic representations of linear algebraic groups has been developed by P. Schneider, U. Stuhler, M. Rapoport et al.

Chapter 9 aims to present the rigid analytic part of Raynaud's proof of the Abhyankar conjecture for the affine line, with just rudiments of that theory. Basically, only a small part of Chapter 2 is needed. For completeness, other parts of Raynaud's proof together with work of J.-P. Serre, D. Harbater and F. Pop are discussed. The "References" makes no pretense of being complete.

Finally, we have tried to be careful in the references to authors and apologize beforehand for possible mistakes in this respect.

List of conventions. We shall use the letters $\mathbf{C}, \mathbf{N}, \mathbf{Q}, \mathbf{R}, \mathbf{Z}$ for the complex numbers, the non-negative integers, the rational numbers, the real numbers and the integers, respectively. For any ring R, the group of the invertible elements of R is written as R^*. The separable algebraic closure and the algebraic closure of a field F are denoted by F^{sep} and F^{alg}. For a field F provided with a non-trivial non-archimedean valuation $|\ |$ we write $\widehat{F}$ for the completion of F. The symbols $F^o, F^{oo}, \bar{F}$ stand for the valuation ring of F, its maximal ideal and the residue field. $\mathbf{Q}_p$ and $\mathbf{C}_p$ will denote the field of the p-adic numbers and the completion of the algebraic closure of the field of the p-adic numbers. For any power q of a prime number, $\mathbf{F}_q$ denotes the field with q elements. The letter k will (in general) denote a field that is complete with respect to a non-trivial non-archimedean valuation. The letter K will (in general) denote an algebraically closed field that is complete with respect to a non-trivial non-archimedean valuation. The expressions $|\ |$ and $\|\ \|$ stand for the valuation and the norm.

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