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S-Variable Approach to LMI-Based Robust Control

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Preface

At the end of 1990s, we discovered with enthusiasm the publication by Mauricio C. de Oliveira, Jacques Bernussou, and José C. Geromel concerning a "New Discrete-Time Stability Condition." Since then, we have dedicated most of our attention to understanding and applying the underlying powerful methodology. We have extended the results in many directions and witnessed major interest of our colleagues for these extensions. The technique is now widely used but never was completely formalized. Because of that, it has been sometimes missused or missinterpreted, some central issues have not been clarified, and there is not even a unique denomination for it. In this book, we decide to call that methodology the *S*-variable approach, following the denomination *S*-procedure that is a powerful tool in dealing with constrained inequality conditions. We explain this choice, expose in details our understanding of it, and provide important robust control results related to it.

The *S*-variable approach, or SV approach for short, enters the well developed field of convex-optimization-based control theory. Since the pioneering results on linear matrix inequality (LMI) and semidefinite programming (SDP) in the 1980s, convex optimization has become a standard strategy for system analysis and synthesis. Not only existing closed-form analytic solutions can be losslessly reformulated as the feasibility of SDPs, efficient polynomial-time powerful softwares are available, but the convex optimization point of view also allows the derivation of new major results, especially for robust control.

Even though the convex optimization approach to robust control problems has provided answers to many practical problems, these answers happen to be, except in few simple cases, "conservative." The control problems can in general be formulated as finding some control (defined as a gain or a dynamic equation) that guarantees one or several performances for the controlled system (some physical processes most often conceived by engineers, but it could as well be an economic or an ecologic system). To perform the design one assumes a mathematical model for the system. This model is imperfect and robust control aims at guaranteeing the performances whatever bounded model uncertainties. The design thus formulates as a minimax problem: finding the best controller against the worst-case uncertainty.

The exact LMI characterization of most such robust control problems happens to be out of range, but LMIs do provide upper-bounds. The gap with the exact value characterizes the conservatism of the result. LMI results issued from the SV approach prove theoretically to reduce this conservatism, and do reduce largely the gap in practice. The conservatism reduction provided by the SV approach is the central property discussed in the book.

The basics of the SV approach can be seen immediately when considering a simple example. Let a linear time-invariant system be described in state-space by the differential equation $\dot{x}(t) = Ax(t)$. Its asymptotic stability (i.e., the convergence of the state $x(t)$ to zero as times goes to infinity) is ensured by the existence of a positive quadratic function $V(x) = x^T Px$, called a *Lyapunov function*, such that its derivatives are negative along all trajectories of the system:

$$\dot{V}(x) \;=\; \dot{x}^T Px + x^T P\dot{x} < 0 \quad \forall (x, \dot{x}) \text{ such that } \dot{x} = Ax,\ x \neq 0.$$

LMI formulas to that problem are derived by including the equality constraint into the inequality constraint. The first classical way of doing so is to replace $\dot{x}$ by its value Ax. This leads to the following matrix valued inequalities

$$P \succ 0, \quad A^T P + PA \prec 0$$

where the $\succ 0$ ($\prec 0$) stands for "positive (negative) definiteness." These matrix inequalities are linear with respect to the decision variable P, it is an LMI feasibility problem. On the other hand, the second way is rewrite the condition explicitly as a constrained inequality condition as in

$$\begin{bmatrix} \dot{x} \\ x \end{bmatrix}^T \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix}^T < 0 \quad \forall \begin{bmatrix} \dot{x} \\ x \end{bmatrix} \text{ such that } \begin{bmatrix} I & -A \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix} = 0,\ x \neq 0.$$

By closely following the idea of S-procedure, we obtain

$$P \succ 0, \begin{bmatrix} 0 & P \\ \mathrm{P} & 0 \end{bmatrix} + F[1 - A] + \begin{bmatrix} I \\ A^T \end{bmatrix} F^T \prec 0$$

where the decision variables are now P and F. The second formulation corresponds to the SV approach and the LMIs will be called SV-LMIs. The two LMI results are of course mathematically equivalent in the sense that the former is feasible if and only if the latter is feasible. However, in the SV-LMIs the matrix F introduces additional degrees of freedom and somehow relaxes or dilates the inequalities. This matrix F is called the S-variable, following the denomination S-procedure. The book is dedicated to exploring the benefits (and the drawbacks) of this approach, shortened by SV-approach.

The main drawback to be mentioned all at once is in terms of numerical computation. The size of the SV-LMI problem roughly doubles (both in terms of size of the inequalities and of the number of variables). Therefore, one can expect increased computation time when solving SV-LMIs with available SDP solvers. But, can conservatism reduction be obtained for free?

The main advantage is the decoupling SV produces between the decision variable and the data (P and A, respectively, in the above example). This decoupling is the key for conservatism reduction and new powerful robust analysis results follow from it. Chapters 2, 3 and 7 of the book are dedicated to these robust analysis issues (computing upper bounds on worst case performances for a given control). But as soon as the control design is considered the decoupling is no more absolute. In the above example, the design problem amounts to replacing A by a controller dependent matrix $A(K)$, thus creating couplings between the controller and the S-variable (K and F, respectively). Handling this issue while remaining in an LMI context and keeping the conservatism reduction is the central question of SV-LMI-based control design. Chapters 4–6 and 8 of the book are dedicated to this question.

The more precise scope of the monograph is now presented by outlining each chapter contents.

Chapter 1 is dedicated to the origins of the SV approach. We trace the contributions of independent authors that participated to establish the fundamentals and justify our choice for the denomination "S-variable approach". This attentive detailed study is the occasion to show several interpretations of the S-variables with respect to technical results such as Finsler's lemma and elimination lemma. The chapter concludes with the brief exposure of all the problems to be tackled in the remaining part of the book, justifying the importance of these selected problems.

The primary goal of Chap. 2 is founding the basic idea of the SV approach. Before generalizing the technique (which is done in the following chapters) we show its effectiveness on simple essential control problems. We mainly consider the robust performance analysis problems of linear time-invariant systems affected by parametric uncertainties, and clarify why the SV-LMIs do perform well on these intractable infinite-dimensional semi-infinite problems. We also highlight the improvements in terms of conservatism both theoretically and on examples.

Chapter 3 is an extension of the SV-LMIs of the preceding chapter both in terms of generalization of the results and for further conservatism reduction. First the SV results are generalized to descriptor systems. Not only this result is valuable in itself, but also combined to a model manipulation technique, where an infinite sequence of SV-LMIs can be built. This sequence of SV-LMIs is shown to be easy to construct and proved to be of decreasing conservatism. Tests are provided for checking if the conservatism gap vanishes. With examples it is shown that the conservatism gap indeed vanishes, and this is obtained by early elements of the sequence (i.e., the convergence is rather fast). The chapter concludes with the mathematical description of SV approach for a class of robust LMI problems.

In Chap. 4 the SV-LMIs of Chap. 2 are reconsidered for robust state-feedback design. The results rely on the structuring of the S-variables. An interpretation in terms of virtual stable model is given to this structure. Moreover, we show the effect of this structuring on conservatism reduction. It happens to be of different nature in the discrete-time and continuous-time cases.

Chapter 5 focuses on multi-objective control design. The goal here is to design a controller that satisfy multiple design specifications. In the state-feedback case, the advantage of the SV-LMIs readily follows from the results in Chap. 4. To handle output-feedback case, we first extend the results of Chap. 4 and provide SV-LMI-based formulas for dynamic output feedback controller synthesis. Then, the effectiveness of the SV-LMIs in conservatism reduction can be shown almost the same way as in the state-feedback case.

While the two previous chapters address control design cases, which have LMI solutions, Chap. 6 tackles the hard problem of static-output feedback design. No polynomial-time optimization method exists for that problem, and if keeping in the matrix inequality framework, one has to resort to iterative LMI algorithms. For that important and hard problem the SV approach produces an sophisticated version of such iterative LMI algorithm. In this case, the structuring of the *S*-variables reveals a virtual stabilizing state-feedback. This original design procedure is detailed and tested on examples.

Finally, Chaps. 7 and 8 are dedicated to the case of discrete-time periodic systems. For that special case the SV-LMIs have interesting noncausal system interpretations in analysis (Chap. 7). Similarly to the LTI case, SV-LMIs are effective in reducing the conservatism of the analysis and synthesis results when dealing with discrete-time periodic systems affected by polytopic uncertainties.

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Contents

Notations

Some Specific Sets

$\mathbb{R}$	Real numbers
$\mathbb{R}^n$	Real n-vectors ($n \times 1$ matrices)
$\mathbb{R}^{m\times n}$	Real $m \times n$ matrices
$\mathbb{C}$	Complex numbers
$\mathbb{C}^n$	Complex n-vectors
$\mathbb{C}^{m\times n}$	Complex $m \times n$ matrices
$\mathbb{C}_-$	Complex numbers with strictly negative real part
$\mathbb{D}$	Open unit disc on the complex plane
$\partial\mathbb{D}$	Unit circle on the complex plane
$\mathbb{S}^n$	Real symmetric $n \times n$ matrices
$\mathbb{S}^n_+$, $\mathbb{S}^n_{++}$	Real symmetric positive semidefinite, positive definite, $n \times n$ matrices
$\mathbb{H}^n$	Complex Hermitian $n \times n$ matrices
$\mathbb{H}^n_+$, $\mathbb{H}^n_{++}$	Complex Hermitian positive semidefinite, positive definite, $n \times n$ matrices
$\lambda(A)$	The set of the eigenvalues of $A \in \mathbb{C}^{n\times n}$
$\mathscr{I}_{n_1,n_2}, \mathscr{I}_n$	Positive integers from n_1 to n_2 and 1 to n, i.e., $\mathscr{I}_{n_1,n_2} := \{n_1, \cdots, n_2\}$ and $\mathscr{I}_n := \{1, \cdots, n\}$.

Vectors and Matrices

$\mathbf{1}(\mathbf{1}_n)$	Vector with all components one (of size n)
x_i	i-th element of vector x
e_i	i-th standard basis vector
I (I_n)	Identity matrix (of size n)
X^T	Transpose of matrix X
X^*	Complex conjugate transpose of matrix X

trace (X)	Trace of matrix X
rank (X)	Rank of matrix X
	The following two notations assume $X \in \mathbb{R}^{n\times m}$ and rank $(X) = r$
$X^{\perp}$	Full rank matrix such that $X^{\perp} \in \mathbb{R}^{(n-r)\times n}$, $X^{\perp}X = 0$ (the image of X is the kernel of $X^{\perp}$, `(null(X'))'` in Matlab©)
X°	Full rank matrix such that $X^{\circ} \in \mathbb{R}^{m\times r}$, XX° is full rank (the image of X° is orthogonal to the kernel of X, `orth(X')` in Matlab©).

Norms for Vectors, Matrices and Signals

$\|x\|$	Euclidean norm of vector x
$\|X\|$	Maximum singular value of matrix X
$\|z\|_2$	L_2 norm of the signal $z(t)$, i.e., $\| z \|_2 := \sqrt{\int_0^{\infty} \| z(t) \|^2 \, dt}$

Inequalities

$x \geq y$	Elementwise inequality between real vectors x and y
$x > y$	Strict elementwise inequality between real vectors x and y
$X \succeq Y$	Matrix inequality between real symmetric matrices X and Y
$X \succ Y$	Strict matrix inequality between real symmetric matrices X and Y

Norms for Stable LTI Systems

$\|G\|_2$	H_2 norm of stable LTI system G
$\|G\|_{\infty}$	H_{∞} norm of stable LTI system G

Functions

$\mathrm{He} : \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$	$\mathrm{He}(X) := X + X^T$
$\mathrm{Sq} : \mathbb{R}^{m\times m} \to \mathbb{R}^{m\times m}$	$\mathrm{Sq}(X) := XX^T$

Simplex

Let us define

$$\mathbb{E}^n := \{\theta \in \mathbb{R} \, : \, \theta \geq 0, \mathbf{1}^T\theta = 1\}$$

Then, $\mathbb{E}^n$ is said to be a standard simplex on $\mathbb{R}^n$.

Polytopes

For given matrices $A^{[j]} \in \mathbb{R}^{n\times n}$ ($j \in \mathscr{I}_L$), let us define $A := \{A^{[j]}, \cdots, A^{[L]}\}$. Then, the convex hull of the set A, denoted by $\text{conv}(A)$, is defined by

$$\text{conv}(A) := \left\{ \sum_{j=1}^{L} \theta_j A^{[j]} \ : \ \theta \in \mathbb{E}^L \right\}$$

In the control literature, $\text{conv}(A)$ is also called as a polytope with L vertex matrices $A^{[j]}$ ($j \in \mathscr{I}_L$).