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An Introduction to Laplace Transforms and Fourier Series

Second Edition

Springer

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To Ottilie

Preface

This book has been primarily written for the student of mathematics who is in the second year or the early part of the third year of an undergraduate course. It will also be very useful for students of engineering and physical sciences for whom Laplace transforms continue to be an extremely useful tool. The book demands no more than an elementary knowledge of calculus and linear algebra of the type found in many first year mathematics modules for applied subjects. For mathematics majors and specialists, it is not the mathematics that will be challenging but the applications to the real world. The author is in the privileged position of having spent ten or so years outside mathematics in an engineering environment where the Laplace transform is used in anger to solve real problems, as well as spending rather more years within mathematics where accuracy and logic are of primary importance. This book is written unashamedly from the point of view of the applied mathematician.

The Laplace transform has a rather strange place in mathematics. There is no doubt that it is a topic worthy of study by applied mathematicians who have one eye on the wealth of applications; indeed it is often called Operational Calculus. However, because it can be thought of as specialist, it is often absent from the core of mathematics degrees, turning up as a topic in the second half of the second year when it comes in handy as a tool for solving certain breeds of differential equation. On the other hand, students of engineering (particularly the electrical and control variety) often meet Laplace transforms early in the first year and use them to solve engineering problems. It is for this kind of application that software packages (MATLAB©, for example) have been developed. These students are not expected to understand the theoretical basis of Laplace transforms. What I have attempted here is a mathematical look at the Laplace transform that demands no more of the reader than a knowledge of elementary calculus. The Laplace transform is seen in its typical guise as a handy tool for solving practical mathematical problems but, in addition, it is also seen as a particularly good vehicle for exhibiting fundamental ideas such as a mapping, linearity, an operator, a kernel and an image. These basic principals are covered in the first three chapters of the book. Alongside the Laplace transform, we develop the notion of Fourier series from first principals. Again no more than a working knowledge of trigonometry and elementary calculus is required from the student. Fourier series can be introduced via linear spaces, and exhibit properties such as orthogonality, linear independence and completeness which are so central to much of mathematics. This pure mathematics would be out of place in a text such as this, but Appendix C contains much of the background for those interested. In Chapter 4, Fourier series are introduced with an eye on the practical applications. Nevertheless it is still useful for the student to have encountered the notion of a vector space before tackling this chapter. Chapter 5 uses both Laplace transforms and Fourier series to solve partial differential equations. In Chapter 6, Fourier Transforms are discussed in their own right, and the link between these, Laplace transforms and Fourier series, is established. Finally, complex variable methods are introduced and used in the last chapter. Enough basic complex variable theory to understand the inversion of Laplace transforms is given here, but in order for Chapter 7 to be fully appreciated, the student will already need to have a working knowledge of complex variable theory before embarking on it. There are plenty of sophisticated software packages around these days, many of which will carry out Laplace transform integrals, the inverse, Fourier series and Fourier transforms. In solving real-life problems, the student will of course use one or more of these. However, this text introduces the basics; as necessary as a knowledge of arithmetic is to the proper use of a calculator.

At every age there are complaints from teachers that students in some respects fall short of the calibre once attained. In this present era, those who teach mathematics in higher education complain long and hard about the lack of stamina amongst today's students. If a problem does not come out in a few lines, the majority give up. I suppose the main cause of this is the computer/video age in which we live, in which amazing eye-catching images are available at the touch of a button. However, another contributory factor must be the decrease in the time devoted to algebraic manipulation, manipulating fractions etc. in mathematics in the 11–16 age range. Fortunately, the impact of this on the teaching of Laplace transforms and Fourier series is perhaps less than its impact in other areas of mathematics. (One thinks of mechanics and differential equations as areas where it will be greater.) Having said all this, the student is certainly encouraged to make use of good computer algebra packages (e.g. MAPLE©, MATHEMATICA©, DERIVE©, MACSYMA©) where appropriate. Of course, it is dangerous to rely totally on such software in much the same way as the existence of a good spell checker is no excuse for giving up the knowledge of being able to spell, but a good computer algebra package can facilitate factorisation, evaluation of expressions, performing long winded but otherwise routine calculus and algebra. The proviso is always that students must *understand* what they are doing before using packages as even modern day computers can still be extraordinarily dumb!

In writing this book, the author has made use of many previous works on the subject as well as unpublished lecture notes and examples. It is very difficult to know the precise source of examples especially when one has taught the material to students for some years, but the major sources can be found in the bibliography. I thank an anonymous referee for making many helpful suggestions. It is also a great pleasure to thank my daughter Ottilie whose familiarity and expertise with certain software was much appreciated and it is she who has produced many of the diagrams. The text itself has been produced using LATEX.

January 1999

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Preface to the Second Edition

Twelve years have elapsed since the first edition of this book, but a subject like Laplace transforms does not date. All of the book remains as relevant as it was at the turn of the millennium. I have taken the opportunity to correct annoying typing errors and other misprints. I would like to take this opportunity to thank everyone who has told me of the mistakes, especially those in the 1999 edition many of which owed a lot to the distraction of my duties as Head of School as well as my inexperience with LATEX. Here are the changes made; I have added a section on generalising Fourier series to the end of Chap. 4 and made slight alterations to Chap. 6 due to the presence of a new Chap. 7 on Wavelets and Signal Processing. The changes have developed both out of using the book as material for a second-year module in Mathematical Methods to year two undergraduate mathematicians for the past 6 years, and the increasing importance of digital signal processing. The end of the chapter exercises particularly those in the early chapters have undergone the equivalent of a good road test and have been improved accordingly. I have also lengthened Appendix B, the table of Laplace transforms, which looked thin in the first edition.

The biggest change from the first edition is of course the inclusion of the extra chapter. Although wavelets date from the early 1980s, their use only blossomed in the 1990s and did not form part of the typical undergraduate curriculum at the time of the first edition. Indeed the texts on wavelets I have quoted here in the bibliography are securely at graduate level, there are no others. What I have done is to introduce the idea of a wavelet (which is a pulse in time, zero outside a short range) and use Fourier methods to analyse it. The concepts involved sit nicely in a book at this level if treated as an application of Fourier series and transforms. I have not gone on to cover discrete transforms as this would move too far into signal processing and require statistical concepts that would be out of place to include here. The new chapter has been placed between Fourier Transforms (Chap. 6) and Complex Variables and Laplace Transforms (now Chap. 8).

In revising the rest of the book, I have made small additions but no subtractions, so the total length has increased a little.

Finally a word about software. I have resisted the inclusion of pseudocode or specific insets in MATLAB or MAPLE, even though the temptation was strong in relation to the new material on wavelets which owes its popularity largely to its widespread use in signal processing software. It remains my view that not only do these date quickly, but at this level the underlying principles covered here are best done without such embellishments. I use MAPLE and it is updated every year; it is now easy to use it in a cut and paste way, without code, to apply to Fourier series problems. It is a little more difficult (but not prohibitively so) to use cut and paste methods for Laplace and Fourier transforms calculations. Most students use software tools without fuss these days; so to overdo the specific references to software in a mathematics text now is a bit like making too many specific references to pencil and paper 50 years ago.

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