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Sampled-Data Models for Linear and Nonlinear Systems

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To Paz and Rosslyn

Preface

Most real-world systems evolve in continuous time. However, digital implementation is almost universally used in practice. Hence, a crucial ingredient in practical estimation and control is an understanding of the impact of sampling on continuous-time models and systems. In this context, the aim of this book is to reduce the gap between continuous-time and sampled-data systems theory. The subject of sampling is huge—no one book can cover all aspects. Thus, the book emphasises exact and approximate models for sampled-data systems. Questions such as the following will be addressed:

- *What can one say when the sampling rate is high relative to the dynamics of interest?*
- *Do natural convergence results apply as the sampling rate increases?*
- *Do there remain any special features of sampled systems which are not associated with underlying continuous systems?*

The authors' motivation for writing the book was threefold:

(i) Whilst most systems evolve in continuous time, all modern control and signal processing equipment is computer based. Hence, sampling arises as an inescapable aspect of all modern control and signal processing applications.
(ii) Sampling is, at first glance, a straightforward issue. However, on closer examination, if sampling is not treated properly, misleading or erroneous results can occur.
(iii) The authors have found that many aspects of sampling are not completely understood by engineers and scientists, even though these issues are central to many of the applications with which they deal.

The goal of this book is to provide a guide for students, practising engineers, and scientists who deal with sampled-data models. The book is intended to act as a catalyst for further applications in the area of nonlinear estimation and control.

Four classes of systems are treated:

(i) linear deterministic systems,
(ii) nonlinear deterministic systems,

(iii) linear stochastic systems, and
(iv) nonlinear stochastic systems.

Several applications are also presented. These applications embellish the core ideas by showing how they impact several important problems in signals and systems.

The book was written in Valparaíso (Chile) and Newcastle (Australia) during enjoyable collaborative visits by the authors. The book assembles contemporary work of the authors and others on sampled-data models. The authors hope that, by setting these ideas down in one place, we can instill confidence in readers dealing with sampled-data issues in real-world applications of signal processing and control.

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Valparaíso, Chile
Newcastle, Australia
September 19, 2013

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Part II Stochastic Systems

Symbols and Acronyms

$\langle \circ, \circ \rangle$	Inner product.
$\sim$	*Distributed as* (for random variables).
*	Complex conjugation.
^{T}	Matrix (or vector) transpose.
γ	Complex variable associated to the δ-operator.
Δ	Sampling period.
δ	Delta operator (forward divided difference).
$\delta(t)$	Dirac delta or continuous-time impulse function.
$\delta_K[k]$	Kronecker delta or discrete-time impulse function.
$\mu(t)$	Unitary step function or Heaviside function.
$\mu[k]$	Discrete-time unitary step function.
$\rho = \frac{d}{dt}$	Time-derivative operator.
ω	Angular frequency, in [rad/s].
ω_N	Nyquist frequency, $\omega_N = \frac{\omega_s}{2}$.
ω_s	Sampling frequency, $\omega_s = \frac{2\pi}{\Delta}$.
A, B, C, D	State-space matrices in continuous time.
$A_\delta, B_\delta, C_\delta, D_\delta$	State-space matrices in discrete time using the δ-operator (i.e., incremental models).
A_q, B_q, C_q, D_q	State-space matrices in discrete time using the shift operator q.
adj	Adjoint of a matrix.
ASZ	Asymptotic sampling zeros.
$\mathbb{C}$	Set of complex numbers.
$\mathcal{C}^n$	Space of functions whose first n derivatives are continuous.
CAR	Continuous-time auto regressive.
CSZ	Corrected sampling zeros.
CT	Continuous time.
CTWN	Continuous-time white noise.
det	Determinant of a matrix.
DFT	Discrete Fourier transform.
DT	Discrete time.
DTFT	Discrete-time Fourier transform.

DTWN	Discrete-time white noise.
$E\{\cdot\}$	Expected value.
ESD	Exact sampled-data (model).
$f(t)$	Continuous-time signal ($t \in \mathbb{R}$).
f_k or $f[k]$	Discrete-time signal or sequence ($k \in \mathbb{N}$).
$\mathcal{F}\{\cdot\}$	(Continuous-time) Fourier transform.
$\mathcal{F}^{-1}\{\cdot\}$	(Continuous-time) inverse Fourier transform.
$\mathcal{F}_d\{\cdot\}$	Discrete-time Fourier transform.
$\mathcal{F}_d^{-1}\{\cdot\}$	Discrete-time inverse Fourier transform.
FDML	Frequency-domain maximum likelihood.
FOH	First order hold.
$G(\rho)$	Deterministic part of a continuous-time system.
$G(s)$	Continuous-time transfer function (s-domain of the Laplace transform).
$G_\delta(\gamma)$	Discrete-time transfer function (γ-domain of the δ operator).
$G_q(z)$	Discrete-time transfer function (z-domain of the shift operator q).
GHF	Generalized hold function.
GSF	Generalized sampling filter.
$H(\rho)$	Stochastic part of a continuous-time system.
$h_g(t)$	Impulse response of a generalized hold or sampling function.
$\Im$	Imaginary part of a complex number.
I_n	Identity matrix of dimension n.
IV	Instrumental variables.
$\mathcal{L}\{\cdot\}$	Laplace transform.
$\mathcal{L}^{-1}\{\cdot\}$	Inverse Laplace transform.
ℓ_2	Space of square summable sequences.
$\mathcal{L}_2$	Space of square integrable functions.
LQ	Linear-quadratic (optimal control problem).
LS	Least squares.
MIFZ(D)	Model incorporating fixed zero (dynamics).
MIMO	Multiple-input multiple-output (system).
MIPZ(D)	Model incorporating parameterised zero (dynamics).
ML	Maximum likelihood.
MPC	Model predictive control.
$\mathbb{N}$	Set of natural numbers (positive integers).
$N(\mu, \sigma^2)$	Normal (or Gaussian) distribution, with mean μ and variance σ^2.
NMP	Non-minimum phase.
$\mathcal{O}(\Delta^n)$	Function *of order* Δ^n.
PEM	Prediction error method(s).
PSD	Power spectral density.
QP	Quadratic programming.
q	Forward shift operator.
$\Re$	Real part of a complex number.

$\mathbb{R}$	Set of real numbers.
s	Complex variable corresponding to the Laplace transform.
SD	Sampled data.
SDE	Stochastic differential equation.
SDR (or **SDRM**)	Simple derivative replacement (model).
SISO	Single-input single-output (system).
TTS	Truncated Taylor series.
$\mathbb{Z}$	Set of integer numbers.
$\mathcal{Z}\{\cdot\}$	$\mathcal{Z}$-transform.
$\mathcal{Z}^{-1}\{\cdot\}$	Inverse $\mathcal{Z}$-transform.
z	Complex variable corresponding to the $\mathcal{Z}$-transform of the shift operator q.