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Frequency Domain Criteria for Absolute Stability

A Delay-Integral-Quadratic Constraints Approach

Springer

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To Professor Vladimir Andreevich Yakubovich, my lifelong mentor

Preface

The problem of absolute stability has a very long history from its original publication in 1944 by Lurye[1] and Postnikov. Nevertheless, the interest in this problem among control systems theorists is not diminishing. On the contrary, it is increasing. Moreover, some of the new problems, such as robustness and control of systems with uncertainties, can in some sense be viewed as reformulations of this classical problem.

Roughly speaking, the problem is concerned with stability of systems consisting of a linear and a nonlinear block. Only partial information about the latter is given, and stability criteria must involve only the linear block and the known properties of the nonlinear one.

The goal of this book is to bring together some of the most significant results that appeared recently in journals and conference proceedings. Particular attention is paid to the relatively new method, known as delay-integral-quadratic constraints. It turns out that this method sheds some additional insights on a few classical results and also makes it possible to extend them to new classes of systems. Specifically, a number of stability criteria, known previously for autonomous systems, can be extended to time-dependent and, in particular, periodic cases.

As the title of the book suggests, the results are presented in frequency domain, the form in which they tend to naturally arise. In most cases, frequency-domain criteria can be converted to computationally tractable linear matrix inequalities. However, in some cases, inferences concerning system stability can be made directly from the frequency-domain inequalities, especially those that have a certain geometric interpretation, which we discuss in more detail than many other books on the subject.

The book is written in the traditional "theorem-proof" format (except for Chap. 1). However, it is hoped that it could be read by a control systems engineer having a standard background in linear control systems and a certain level of "mathematical maturity." Some of the more technically difficult proofs are given in separate sections at the end of chapters. These sections can be skipped without loss of continuity.

Some of the necessary prerequisites that may be unfamiliar to a control systems engineer are reviewed in the Appendix. The goal of this Appendix is not to give a rigorous theory but rather to "demystify" such concepts as measure, Lebesgue integration, and function spaces. These concepts are used extensively in the formulations of the stability criteria.

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[1] I believe this is the correct transliteration of the Russian name. Other transliterations that have been used are "Lur'e," "Lurie," and "Lure."

The monograph is organized as follows.

Chap. 1 is a historical survey. While there are many surveys of the problem of absolute stability, greater focus is given to the frequency-domain methods, from the classical Popov criterion, through the development of the method of integral-quadratic constraints. Major milestones are reviewed and some applications are discussed.

Mathematical foundations are laid out in Chap. 2. They include the so-called quadratic criterion for absolute stability and some integral inequalities, the concepts used throughout the book to prove the main results.

Chap. 3 focuses on the so-called stability multipliers. A generalization of the classical result of Zames and Falb is presented. This generalization turns out to be applicable to nonstationary systems. In addition, we discuss a geometric interpretation of stability multipliers, which apparently has not yet been discussed in monographic literature, only in Russian journal publications in the 1980s.

In Chap. 4, the attention is shifted to time-periodic systems. It turns out that stability multipliers, discussed in Chap. 3, take a certain specific form when the nonlinear block is time-periodic. Furthermore, the geometric interpretation discussed in Chap. 3 can also be applicable. The chapter concludes with discussion of some open problems.

This book is an expanded version of my Ph.D. dissertation presented to the Department of Theoretical Cybernetics of St. Petersburg State University (Russia). Many of these ideas were developed under the guidance of my long-time mentor, Professor Vladimir Andreevich Yakubovich. It is to him that I dedicate this work with deep gratitude.

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