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Iterative Methods for Queuing and Manufacturing Systems

With 17 Figures



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To Mandy Lee, Shun Tai Ching,
and Kam Koo Wong

Preface

The aim of this book is to outline the recent development of iterative methods for solving Markovian queuing and manufacturing systems. Markov processes are widely used in the modeling of queuing systems, manufacturing systems, telecommunication systems, computer systems, and many other practical systems. Very often, in the system performance analysis, one faces the problem of solving the system steady-state probability distribution of a large number of states. Classical iterative methods such as Gauss-Seidel (GS) and Jacobi methods are the common iterative methods used to tackle the problem. However, their convergence rates are slow in general and very often increase linearly with respect to the number of states in the system. In this book, we consider Conjugate Gradient (CG) type methods in solving the steady-state probability distribution of queuing and manufacturing systems. Usually, the CG method is used with a matrix, called a preconditioner, to accelerate its convergence rate. We construct preconditioners for Markovian queuing and manufacturing systems such that the Preconditioned Conjugate Gradient (PCG) method converges very fast. The construction techniques can be applied to a large class of practical systems. Apart from computational methods, interesting Markovian models with applications in inventory control and supply chains are also introduced.

This book consists of nine chapters. Chapter 1 gives a brief introduction, with overviews of Markov processes, simple queuing and manufacturing systems, and iterative methods.

In Chapter 2, we discuss Toeplitz-circulant preconditioners for queuing systems with batch arrivals. In Chapter 3, we present a queuing system with Markov Modulated Poisson Process (MMPP) inputs, which arise in telecommunication systems. The CG method is applied to solve the steady-state probability distribution with the preconditioner constructed by taking a circulant approximation of the generator matrix.

In Chapter 4, we discuss an application of MMPP to the manufacturing systems of multiple failure-prone machines under Hedging Point Production (HPP) policy. In Chapter 5, we consider failure-prone manufacturing systems with batch arrivals of demand. A preconditioner is constructed by exploiting the structure of the generator matrix of the system.

In Chapter 6, we model a flexible manufacturing system by using a Markovian queue. In Chapter 7, we discuss a manufacturing system of two machines in tandem. HPP policy is employed as the production control. The steady-state probability distribution of the system is then solved by the CG method with a preconditioner constructed by taking circulant approximation of the generator matrix.

In Chapter 8, we discuss an analytical model for manufacturing systems under delivery time guarantee policy. An analytical solution of the steady-state probability distribution of the inventory system is obtained and is applied to the analysis of the model. In Chapter 9, we discuss an application of MMPP in modeling multi-location inventory systems. A Markovian queuing model for supply chains problems is also introduced.

This book is aimed at students, professionals, practitioners, and researchers in applied mathematics, scientific computing, and operational research, who are interested in the formulation and computation of queuing and manufacturing systems. Readers are expected to have some basic knowledge of Markov processes and matrix theory.

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All the figures, tables, propositions and lemmas in Chapters 2, 3 and 4 are reprinted from [25, 50] with permission from SIAM. All the figures, tables, propositions and lemmas in Chapters 5, 7, 8 and 9 are reprinted from [35, 36, 40, 41] with the permission from Elsevier Science.

Southampton, December 2000

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