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E.G. Peter Rowe

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Foreword

Dr E.G. Peter Rowe had almost completed writing this book when his life was brutally terminated in the Yemen in December 1998.

Peter came to Durham from Canada via London in 1964 and quickly became a very popular and engagingly eccentric member of our Department. He was friendly and generous, informal, charitable, warm and full of diverse interests which, apart from mathematics and physics, ranged from anthropology and archaeology to foreign cultures and travel. This led him to spend his sabbatical leave in places such as northern Nigeria. His interest in other people's cultures took him to many parts of the world (from Ladakh in North West India, to many places in Africa, motorbiking in Saskatchewan, Canada and South Africa and finally to the fatal trip to Yemen).

Peter was witty and was not afraid to speak his mind; at our Departmental meetings we now miss his throw-away but very much to-the-point comments and suggestions.

In his teaching and research Peter was somewhat unconventional. He did not follow fashionable trends in research but worked on what interested him most, namely, the geometrisation of physics.

He considered refereeing of research papers to be an important task, writing long reports full of helpful suggestions to the authors. Peter also took his teaching seriously. His courses, often perceived as difficult by the students, were always quite advanced, as if designed to draw the very best from his audience. His view was that it was better to say something new and stimulating to the interested students than show routine steps to the uninterested ones.

The present book grew out of his course on special and general relativity given to our third year students. I first became aware of Peter's approach to relativity when, having taught a similar course before him, I was asked to check his examination questions. While some of them were routine, others demanded deeper thought and when I looked at his solutions I became aware of the merits of his more geometric approach. I was among those who encouraged him to write a book in order to make this approach available to a wider audience.

Peter's book puts an emphasis on geometry in the description of physical phenomena in Minkowski spacetime. In this it emphasises the covariance properties of the equations of motion, trying as much as possible to avoid working in any particular frame of reference. And the book achieves this aim, probably, more than any other book that I know.

I am very pleased that Springer-Verlag have published Peter's book. The book will not only help many people to understand physics in a more geometrical setting, but also it will be a lasting reminder of our colleague and friend, complementing our personal memories of him.

Wojtek J. Zakrzewski
University of Durham

Preface

This book is **not** meant for the complete beginner in special relativity, **nor** for anyone wanting an account of the numerous and interesting experiments that support the theory. Instead, it is intended to be a description of the geometry of spacetime, and an aid in the creation and development of intuition in four-dimensional Minkowski space. The emphasis on the *geometry* means an emphasis on the *absolutes* which underlie relative descriptions. For example, the Poincaré transformation links different relative sets of coordinates, x^μ, x'^μ, but the underlying absolute is simply a point P in spacetime (the coordinates are the relative descriptions). The deepest understanding, perhaps the only understanding, of relativity and spacetime is in terms of the geometrical absolutes, and this is what the book seeks to develop. Whereas the beginner in special relativity must have help in making the transition between his nonrelativistic view of physics as a time-development in space (his space) to a four-dimensional view of physics as a complete history in spacetime, it is hoped that the reader of this book is ready to study the subject in its final, unified (and beautiful) form.

The mathematical prerequisites for the early chapters of the book are very few, just linear algebra and elementary geometry (done using vectors and a scalar product). For the later chapters multivariable calculus and ordinary differential equations are often needed. No detailed knowledge of the experimental background to relativity is needed, nor any detailed knowledge of electromagnetism, but in both these areas, the more sophistication and sympathy is available for the subjects, the better.

The book aims to cover the most interesting topics requiring special relativity. It is an outgrowth of lectures on special and general relativity given to final year undergraduate students of theoretical physics in the Department of Mathematics. It could be presumed that the students had all had half a dozen or a dozen lectures in earlier years covering the experimental foundations of special relativity and the first, surprising consequences of Einstein's new kinematics. However, the book goes well beyond what was ever taught in practice. Although in a real sense special relativity is the culmination of classical physics, and worthy on that account of detailed study, in the lecture theatre time is limited and the attractions of gravity, with its curved spacetime, become overwhelming. In practice, a natural climax for special relativity is the definition of the energy tensor (which becomes the source of gravitational curvature) and its use in deriving equations of motion. Some of the more difficult aspects of the energy tensor, and most areas of electromagnetism, were left for self study (in the future). The material in the book, therefore, is partly at an undergraduate level and partly at a postgraduate level.

In the first chapter, Spacetime, the idea of a four-dimensional space having special coordinates (arising from the inertial frames of reference) is developed. An attempt is made to distinguish between the mathematical side of the exposition, where clarity and logic can be expected, and the real-world side, still partly unknown and mysterious, where our understanding advances in a series of temporary world views. The present model is described in natural language (not mathematical); it is a familiar world of clocks and spatial frameworks, but mysteriously without gravity. The mathematics we develop is put into correspondence with this model. The Lorentz transformation and the Poincaré transformation are discussed (as distinct from being postulated, or derived from an artificial starting point). The importance of the lightcone in the theory is exemplified by the way it creates a significant division into regions of the spacetime around any given event. In the whole of the chapter, the emphasis is on spacetime and how we can begin to picture events and processes (and inertial frames, which may be *relative* yet are also physical objects) in it.

In the second chapter, the most important one for building intuition in Minkowski space, vectors in spacetime are defined as transformations of points in spacetime (the geometrical or absolute concept), simply expressed in terms of the inertial frames, which both contribute to the definition and provide the relative expressions of the concept. The scalar product of vectors is constructed to provide the vector expression of the division of spacetime determined by the lightcone. All the famous kinematical effects can be given completely transparent discussions in terms of spacetime diagrams and simple vector geometry.

(The first time I taught the course on which this book is based, I attempted to begin with a discussion of vectors in Minkowski spacetime, without any discussion of spacetime as a manifold. Only a few students found this direct approach attractive and were able to build a useful intuition from it. Student discontent resulted in what is now Chapter 1 to fill in all the background material.)

The third chapter, Asymptotic Momentum Conservation, is devoted to the four-momentum of elementary particles and the relations that follow from the simple idea of equating momentum in the past with momentum in the future. All relations can be expressed in purely geometrical terms. The definition of the centre of momentum frame is particularly simple when it is expressed by its geometrically defining property (rather than in terms of its relation with other, irrelevant frames).

In Chapter 4, covectors and dyadics, which are generalisations of vectors, are defined and their properties developed. The gradient of scalar functions in spacetime is defined as a covector, then converted to a vector, then generalised to the gradient of vector fields and beyond. The concept of volume is discussed, as is the divergence theorem in spacetime.

In Chapter 5, the geometrical formulation of electromagnetism is given. The early sections deal with the decomposition of the field dyadic into relative electric and magnetic fields, and the relation of the different expressions of Maxwell's equations. The geometrical discussion of charge density and three-current is given in terms of a model of charged dust. Conservation of charge then has a visualisable form. The electomagnetism of point particles is begun. Because the consideration of point particles involves delta functions, the topic is technically more difficult and may be omitted at a first reading.

The energy tensor is the subject of Chapter 6. The meaning of its different components is developed with the example of flowing dust. Local conservation of four-momentum is expressed by the vanishing of the divergence of the energy tensor. The equation of motion for flowing, charged dust can be derived from this condition. The general definition of the energy tensor can be developed from a Lagrangian in those cases where the equations of motion can be derived from a variational principle.

A point particle with an accelerating timelike worldline creates some special, peculiarly relativistic problems. It is not self-evident how to define the time development of the rest frame. Two solutions, the Fermi-Walker transported frame and the frame which is boosted from the laboratory, both correspond, nonrelativistically, to the unique "nonrotating" frame of Newtonian mechanics. Yet there is a relative rotation between them, the Thomas precession. These problems and their solutions, and the equation of motion of the spin of a point particle with a magnetic moment, are discussed in Chapter 7*.

After every chapter, but especially the first four, are many exercises and problems which supply lots of opportunity to practise the skills and techniques appropriate to special relativistic geometry. And at the very end of each chapter are listed some references for supplementary reading on particular points. No attempt has been made to provide a complete bibliography.

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* Chapter 7 was incomplete at the time of the author's death, and so is not included in the present volume.

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