

# **Parallel Algorithms for Optimal Control of Large Scale Linear Systems**

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# Parallel Algorithms for Optimal Control of Large Scale Linear Systems

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With 35 Figures



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**To Professors: P. Kokotović, H. Khalil  
and J. Medanić**

*Zoran Gajić*

**To my father Zhong Shen**

*Xuemin Shen*

## **Preface**

This book is designed to be a comprehensive treatment of parallel algorithms for optimal control of large scale linear and bilinear systems. These algorithms were originally evolved in the context of the recursive reduced-order methods for singularly perturbed and weakly coupled linear systems. There are numerous examples of large scale singularly perturbed and weakly coupled dynamic systems that provide great challenges to engineers, mathematicians and computer scientists. Some of the examples of singularly perturbed and weakly coupled systems include electric power systems, aerospace systems, large electric and communication networks, robotics, and process control systems in the chemical and petroleum industries.

The parallel algorithms presented in this book are applicable to a wider classes of practical systems than the traditional methods for large scale singularly perturbed and weakly coupled systems based on the power-series expansion methods. The synchronous parallel reduced-order algorithms presented in this book offer several advantages: the higher order of accuracy can be easily achieved at low cost, the parallel processing of information can be used, results are obtained under much milder assumptions (no analyticity requirements imposed on the problem coefficients), the software and hardware implementation of the control algorithms is highly simplified due to complete parallelism in the design procedures.

This book is intended for a wide readership, including control engineers, applied mathematicians, computer scientists, and advanced graduate students who seek a comprehensive view of the current developments in the theory of large scale linear and bilinear singularly perturbed and weakly coupled control systems. The book emphasizes mathematical developments as well as their application to solving practical problems without requiring a strong mathematical background.

To demonstrate the usefulness of the presented methods for large scale singularly perturbed and weakly coupled linear and bilinear systems,

and to point out its various advantages, we have included many real control system examples such as: F-8 aircraft, L-1011 fighter aircraft, fluid catalytic cracker, twelve plate absorption column, magnetic tape control system, power system composed of two interconnected areas, distillation column, steam power system, hydro power plant, chemical plants, gas absorber, supported beam control problem, induction motor drives (bilinear model), large space structure, optimal control of a paper making machine (bilinear model), satellite control problem, and synchronous machine connected to an infinite bus.

The authors hope that this book will reduce some of barriers that exist in recognizing the power and usefulness of the synchronous parallel algorithms for optimal control of large scale linear and bilinear systems, and that it will help to broaden their implementation in practice. Also, we hope that this book will motivate some researchers to develop the corresponding asynchronous parallel algorithms and extend the presented results to nonlinear control systems.

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Z. Gajic and X. Shen

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