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R.B. Bapat

Linear Algebra and Linear Models

Third Edition

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 Springer

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India

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Preface

The main purpose of the present monograph is to provide a rigorous introduction to the basic aspects of the theory of linear estimation and hypothesis testing. The necessary prerequisites in matrices, multivariate normal distribution, and distribution of quadratic forms are developed along the way. The monograph is primarily aimed at advanced undergraduate and first-year master's students taking courses in linear algebra, linear models, multivariate analysis, and design of experiments. It should also be of use to researchers as a source of several standard results and problems.

Some features in which we deviate from the standard textbooks on the subject are as follows.

We deal exclusively with real matrices, and this leads to some nonconventional proofs. One example is the proof of the fact that a symmetric matrix has real eigenvalues. We rely on ranks and determinants a bit more than is done usually. The development in the first two chapters is somewhat different from that in most texts.

It is not the intention to give an extensive introduction to matrix theory. Thus, several standard topics such as various canonical forms and similarity are not found here. We often derive only those results that are explicitly used later. The list of facts in matrix theory that are elementary, elegant, but not covered here is almost endless.

We put a great deal of emphasis on the generalized inverse and its applications. This amounts to avoiding the "geometric" or the "projections" approach that is favored by some authors and taking recourse to a more algebraic approach. Partly as a personal bias, I feel that the geometric approach works well in providing an understanding of why a result should be true but has limitations when it comes to proving the result rigorously.

The first three chapters are devoted to matrix theory, linear estimation, and tests of linear hypotheses, respectively. Chapter 4 collects several results on eigenvalues and singular values that are frequently required in statistics but usually are not proved in statistics texts. This chapter also includes sections on principal components and canonical correlations. Chapter 5 prepares the background for a course in designs, establishing the linear model as the underlying mathematical framework. The sections on optimality may be useful as motivation for further reading in this research area in which there is considerable activity at present. Similarly, the last

chapter tries to provide a glimpse into the richness of a topic in generalized inverses (rank additivity) that has many interesting applications as well.

Several exercises are included, some of which are used in subsequent developments. Hints are provided for a few exercises, whereas reference to the original source is given in some other cases.

I am grateful to Professor Alope Dey, H. Neudecker, K.P.S. Bhaskara Rao, and Dr. N. Eagambaram for their comments on various portions of the manuscript. Thanks are also due to B. Ganeshan for his help in getting the computer printouts at various stages.

About the Second Edition

This is a thoroughly revised and enlarged version of the first edition. Besides correcting the minor mathematical and typographical errors, the following additions have been made:

1. A few problems have been added at the end of each section in the first four chapters. All the chapters now contain some new exercises.
2. Complete solutions or hints are provided to several problems and exercises.
3. Two new sections, one on the “volume of a matrix” and the other on the “star order,” have been added.

About the Third Edition

In this edition the material has been completely reorganized. The linear algebra part is dealt with in the first six chapters. These chapters constitute a first course in linear algebra, suitable for statistics students, or for those looking for a matrix approach to linear algebra.

We have added a chapter on linear mixed models. There is also a new chapter containing additional problems on rank. These problems are not covered in a traditional linear algebra course. However we believe that the elegance of the matrix theoretic approach to linear algebra is clearly brought out by problems on rank and generalized inverse like the ones covered in this chapter.

I thank the numerous individuals who made suggestions for improvement and pointed out corrections in the first two editions. I wish to particularly mention N. Eagambaram and Jeff Stuart for their meticulous comments. I also thank Alope Dey for his comments on a preliminary version of Chap. 9.

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