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Preface

I hope that this book will inspire its readers to like geometry as much as I do. Geometry is the most accessible branch of mathematics. This means that a mind fresh to mathematics can realise its beauty through geometry more quickly than through any other route. When some mathematics students come to university not knowing what an ellipse is, then something is wrong and I hope that this book will go some way towards correcting this. The book is meant to be read by university undergraduates of any year but could be used by a keen sixth former or a research mathematician. The sixth former may not understand everything and the research mathematician may sniff disdainfully at some of the easier passages but I hope that both will find the book useful and perhaps revealing. It is based upon a course of geometry given at Sussex University.

It is pointless and insulting to talk down to anybody so the book contains all levels of mathematics from the easiest, which a lot of readers will have met before, to some quite tricky concepts. I have set the harder parts which can be skipped on a first or second reading in smaller type as follows.

Items in type like this can safely be left to a second or later reading.

In fact I have not followed a slavishly linear route in placing the contents of the book. If the reader wants to read a chapter out of order or feels it worthwhile to dip into another chapter then they can. Some results quoted are too difficult or inappropriate so I have left their proof out. I hope the reader will forgive me.

Some of the displayed mathematics is in bold type. This is to emphasise important results or useful formulæ.

There is no hierarchy or numbering for any results. They are referred to either by name, (e.g. the triangle inequality) or by content, (e.g. the base angles of an equilateral triangle are equal).

▮ Indicates the start of a result and its end is indicated by □

In this book I have taken a very practical point of view and considered the original meaning of *geo-metry* as world-measurement. Ancient peoples needed some descriptive language to measure their fields, count the wheat in a granary, predict astronomical events and do a thousand other calculations of everyday life. They also found in their leisure time that the language of geometry was fun to speculate with and set riddles and puzzles even if the results were not always immediately applicable.

In fact speculation and studying things for their own sake quite often pay off in the future. The early Greek geometers would have been amazed that their ellipses and other conics would have governed the motion of solar system bodies. The eminent mathematician, Hardy, wrote that

"No one has yet discovered any warlike purpose to be served by the theory of numbers..."

However even as his words were first being read, under conditions of great secrecy at Bletchley, the first computer and the fledgling mathematical theory of cryptography was being used to crack the German High Command enigma code. He was to die before the use of huge primes was applied to the coding of signals for the passage of information down a telephone line.

I have tried to set do-able and interesting questions together with revealing examples. Most answers are given and if anyone finds the inevitable mistake please let me know. You can send me the details via my web site

maths.sussex.ac.uk/Staff/RAF/

where mistakes will be noted.

Acknowledgements: Over the years my colleagues, both at Sussex and elsewhere, have freely shared their insights with me for which I am very grateful. My good friends Dale Rolfsen, Colin Rourke and Brian Sanderson are always ready to stimulate my imagination over a glass of something. Robert Smith has often helped my understanding of astronomy. I am particularly grateful to Peter Croyden for keeping my computer going (and Luca Giuzzi). My Ph.D tutor John Reeve was a huge source of geometric ideas. Both Andy Bartholemew and Richard Noss have pointed out typos in the text. Kate Abell and John Claisse have read through the text and questions but final responsibility lies only with me. I am acutely aware that someone else's name should probably also be mentioned and I apologise to them now.

In terms of typesetting, this book would not have been possible without Donald Knuth and his wonderful $\TeX$ program (note, NOT latex). Some people can sit down and write out a book from front to back cover in one continuous stream. However I am not one of those and I need constantly to change and edit any work as it progresses. Without some typesetting program I would have found this impossible and plain $\TeX$ gives you all the flexibility you need to produce beautiful looking mathematics exactly as you want it. I should also put in a word for the program *metapost*, written by J.D. Hobby, which produced the diagrams. It's free, like $\TeX$, and I urge all mathematicians to try it.

Finally I would like to quote from Sir Michael Atiyah's obituary of J.A. Todd, my undergraduate geometry teacher.

> Geometry has always oscillated between the synthetic and the algebraic approach. Diagrams, pictures and mental visions are the heart of the synthetic method, where intrinsic geometric concepts and constructs are the only tools used. By contrast, ever since the time of Descartes, algebraic formulæ and manipulation have provided a mechanical alternative: technically powerful but lacking in insight. As with all fundamental dichotomies, the reality is much more complicated than antagonists allow. Geometrical argument can become lost in its own intricacies, and elegance in algebra can be cultivated. Moreover most new developments involve an appropriate combination of geometric and algebraic ideas, notation and techniques.

Roger Fenn
Sussex, England 2000

In this reprint of the first edition a distressingly large number of typos and errors have been corrected. For this I must thank a number of people but in particular David Brand, Helmer Aslaken, Branko Grünbaum and the students of Dale Rolfsen's geometry class.

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