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Model Reduction for Control System Design

With 39 Figures

Springer

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Preface

The aim of this book is to provide the theoretical foundations of model and controller reduction for students and industrial research and development engineers, especially for those concerned with the application of advanced control theory to actual systems. Readers will obtain a good perspective of recent developments on these problems, and also a grounding in design techniques for practical applications.

This book has its roots in work going back several decades. Modern methods for model reduction perhaps started with a 1966 paper of E. J. Davison. The key feature of the technique advanced in the paper was to retain the dominant modes of the original system and to discard the nondominant modes. This idea is almost the same as a method long used by control engineers, termed dominant pole approximation; however, it was the modern formulation of Davison's paper which stimulated many researchers to move into the field, resulting in an explosion in the development of techniques. In the early work, methods for approximating typical responses of high order were sought. Many papers were published on topics such as minimization of L_2 impulse response error norm, continued fraction approximation, and so on. Then in 1981, B. C. Moore, seeking to introduce the principal component method of statistical analysis into linear dynamical systems, advanced the concept of balanced realizations. This was a major step forward, allowing as it did the introduction of the so-called balanced truncation method for reducing a stable linear system with an associated *a priori* error bound. Some of these reduction schemes are reviewed in Chapter 1 of this book.

Simultaneously with the work on model reduction, work on linear-quadratic-gaussian (LQG) design methods had been progressing and then, beginning in the 1980s, came the developments in H_∞ control design, μ-synthesis and linear matrix inequalities (LMIs). Especially with the latter, it seemed that nearly all linear controller design problems could now be solved. One problem however remained difficult: design of low order controllers for high order plants. The advanced controller design methods tend to supply controllers with order comparable to the plant order, and therefore controllers often of high order. Intuitive understanding of how the controller is functioning and actual implementation in a reliable manner are major tasks with a high order controller, and so a demand arose for techniques of controller order reduction. This cannot be straightforwardly delivered by LMI methods.

The available methods of model reduction did not prove of themselves an effective technique for controller reduction either, since they provide no straightforward way for handling the problem of retaining closed-loop stability and performance when the reduced order controller replaces the full order controller. A moment's reflection shows that if these problems are to be properly addressed, the plant itself has to somehow be brought into the reduction task.

Perhaps the first time this occurred was in a key development of the balanced truncation idea, due to D. F. Enns in 1984 who showed how to capture the plant into the controller reduction problem. His method relied on introducing frequency weighting into the balanced truncation procedure. Controller reduction is the major theme of Chapter 3, when Enns' scheme and other schemes are explored. Another major advance in the model reduction area had come with a famous paper of K. Glover which also appeared in 1984. This paper showed how with state-variable type calculations, a series of approximation problems involving optimal approximation with an unusual norm, the Hankel norm, could be exactly solved. The solutions were also relevant to approximation with more conventional error measures than the Hankel norm. In due course, it was seen how frequency weighting could be introduced into this problem also, so that it could be used as a basis for controller reduction.

The work of Enns and Glover with or without frequency weighting was principally concerned with minimizing a so-called additive error, measured for scalar transfer functions by the difference between two corresponding points on their Nyquist diagrams. Another common form of error between transfer functions can be obtained using the distance between corresponding points on Bode diagrams of amplitude and phase. Focusing on this type of error gives rise to multiplicative or relative error approximation problems, and analogs of balanced truncation approximation and optimal Hankel norm approximation became available for this type of error, starting around the mid 1980s and continuing for almost ten years. K. Glover, M. Green and others developed this type of model reduction, which is described in detail in Chapter 2.

As a further major direction of development occurring principally in the 1990s, we cite the application of coprime fractional descriptions, especially in controller reduction. In Chapter 4 of this book, some techniques of this category are explained, preceded by a short introduction to coprime fractional descriptions.

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The writing of this book was proposed in 1997 by Professor Akira Osumi, of Kyoto Institute of Technology, for one of a Japanese series called the Systems, Control and Information Library, a series sponsored by the Japanese Society for Systems, Control and Information. At that time, Goro Obinata proposed the writing collaboration with Brian Anderson. The writing was carried out in Japanese and English simultaneously, in Akita, Japan and in Canberra, Australia and in two one-week collaborations on the Australian Pacific coast.

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Goro Obinata and Brian D. O. Anderson

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Key Notation

A^T	Transpose of A.
A^*	Complex conjugate transpose of A.
A^+	Pseudo-inverse of A.
$\lambda_i(A)$	Eigenvalue of A.
$\lambda_{\max}(A)$	Eigenvalue of A of maximum modulus.
$\bar{\sigma}(A)$	Maximum singular value of A.
$\sigma_i(G)$	ith Hankel singular value of a stable transfer function matrix $G(s)$.
$G_*(s)$	$G^T(-s)$.
$[G(s)]_+$	Proper part of the strictly stable part of $G(s)$ in an additive decomposition, *i.e.*, the sum of the strictly proper partial fraction summands with poles in the open left half plane plus $G(\infty)$.
$\mathcal{F}_l(\ ,\ )$	Lower Linear Fractional Transformation (explained in text).
RH_∞	Set of stable rational proper transfer function matrices.
RH_∞^-	Set of antistable rational proper transfer function matrices.
$RH_\infty^-(r)$	Set of rational proper transfer function matrices $X(s)$ such that $[X(s)]_+$ has degree at most r.
$S(n_1, n_2)$	Set of rational proper transfer function matrices with n_1 poles in $\mathrm{Re}[s] < 0$ and n_2 poles in $\mathrm{Re}[s] \geq 0$.