LAGRANGE-TYPE FUNCTIONS IN CONSTRAINED NON-CONVEX OPTIMIZATION

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LAGRANGE-TYPE FUNCTIONS IN CONSTRAINED NON-CONVEX OPTIMIZATION

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Preface

Lagrange and penalty function methods provide a powerful approach, both as a theoretical tool and a computational vehicle, for the study of constrained optimization problems. However, for a nonconvex constrained optimization problem, the classical Lagrange primal-dual method may fail to find a minimum as a zero duality gap is not always guaranteed. A large penalty parameter is, in general, required for classical quadratic penalty functions in order that minima of penalty problems are a good approximation to those of the original constrained optimization problems. It is well-known that penalty functions with too large parameters cause an obstacle for numerical implementation. Thus the question arises how to generalize classical Lagrange and penalty functions, in order to obtain an appropriate scheme for reducing constrained optimization problems to unconstrained ones that will be suitable for sufficiently broad classes of optimization problems from both the theoretical and computational viewpoints.

Some approaches for such a scheme are studied in this book. One of them is as follows: an unconstrained problem is constructed, where the objective function is a convolution of the objective and constraint functions of the original problem. While a linear convolution leads to a classical Lagrange function, different kinds of nonlinear convolutions lead to interesting generalizations. We shall call functions that appear as a convolution of the objective function and the constraint functions, *Lagrange-type functions*. We observe that these functions naturally arise as a result of a nonlinear separation of the image set of the problem and a cone in the image-space of the problem under consideration. The class of Lagrange-type functions includes also augmented Lagrangians, corresponding to the so-called canonical dualizing parameterization. However, augmented Lagrangians constructed by means of some general dualizing parameterizations cannot be included in this scheme. We consider them separately.

In a recent corner-stone book [102], an elegant duality theory was developed for an augmented Lagrangian with a convex augmenting function for (nonconvex) optimization problems. However, convexity of augmenting functions for augmented Lagrangians and of convolution functions for Lagrange-type functions sometimes is a restrictive assumption. The following example confirms this: while classical exact penalty functions may not exist for mathematical programs with complementarity constraints, a class of lower order nonconvex and nonsmooth exact penalty functions can be established, see [80]. Our results also confirm that concave convolution functions are better than convex ones for some nonconvex optimization problems, e.g., for a concave minimization problem over a polyhedron set. We study a very general class of abstract Lagrangians, which includes those defined by convex augmenting functions, and level-bounded augmenting functions, as special cases.

The purpose of this book is to provide a systematic examination of Lagrange-type functions and augmented Lagrangians. We will study these functions from three aspects: weak duality, zero duality gap property and the existence of an exact penalty parameter. Weak duality allows one to estimate a global minimum, zero duality gap property allows one to reduce the constrained optimization problem to a sequence of unconstrained problems, and the existence of an exact penalty parameter allows one to solve only one unconstrained problem. By applying Lagrange-type functions, we are able to establish a zero duality gap property for nonconvex constrained optimization problems under a coercive condition. We show that the zero duality gap property is equivalent to the lower semi-continuity of a perturbation function.

The numerical implementation of penalty functions requires the existence of a fairly small exact penalty parameter. This is very important for all local methods and some global methods of Lipschitz programming, otherwise ill-conditioning may occur [34, 42]. We use so-called IPH (increasing positively homogeneous) functions for the convolution of the objective and the constraint functions. Special attention is in particular paid to problems with a single constraint, as optimization problems with multiple constraints can be reduced to such a problem by convoluting all constraints into a single one. For a kind of kth penalty functions we are able to obtain an analytic expression for the least exact penalty parameter. By virtue of this expression we show that the least exact penalty parameter of an kth power penalty function can be diminished if k is small enough. This result leads to a certain reformulation of the initial problem, which allows us to develop and implement a new type of penalty-type functions. These functions can be applied for concave minimization, where the classical penalty function fails.

The outline of the book is as follows.

In Chapter 1, we present motivation for studying Lagrange-type functions, and discuss main questions related to Lagrange-type functions and augmented Lagrangians.

In Chapter 2, we present some auxiliary results related to abstract convexity and theory of IPH functions. The reader can find there a description of the technique, which is used for examination of the zero duality gap property and penalty-type functions for problems with a single constraint.

In Chapter 3, we develop a general scheme of Lagrange-type functions, which is based on a separation of certain sets in the image-space of the problem. We use an elegant idea of Giannessi's as a starting point of our research. Some general results related to weak duality and zero duality gap property are established. The theory of saddle points for Lagrange-type functions, which are linear with respect to the objective function, can also be found there.

In Chapter 4, we consider penalty-type functions for problems with a single constraint using an IPH convolution function. We estimate the least exact penalty parameter for various problems and investigate which reformulation of constrained optimization problems is better from the applications viewpoint. We aim to obtain estimates of parameters, which appear in our approach, and that of the least exact penalty parameter for several classes of problems.

In Chapter 5, we study the zero duality gap property and exactness for various augmented Lagrangian functions, including abstract augmented Lagrangian, level-bounded augmented Lagrangian and sharp augmented Lagrangian.

In Chapter 6, we provide a systematic convergence analysis of optimality conditions of nonlinear penalty-type functions and augmented Lagrangian functions to that of the original constrained optimization problem. The study of approximate solutions and optimal values in terms of Lagrange-type functions is also presented.

Appendix (Chapter 7) contains results of numerical experiments, which confirm that the proposed new nonlinear penalty function works well for some problems of non-convex optimization, including the minimization of a concave function subject to linear constraints.

Some preliminary results on the nonlinear Lagrange-type functions have been included as book chapters in the books 'Abstract Convexity and Global Optimization' [105] by the first author and 'Duality in Optimization and Variational Inequalities' [54] by the second author and his collaborator.

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