Part VI–Nonlinear Optimization

This section introduces you to the analysis of optimization problems whose objectives and constraints can be nonlinear. Even an introductory account of nonlinear programs draws upon material from multi-variable calculus, linear algebra, real analysis, and convex analysis. A coherent account of this background material appears in Chapters 17-19. Nonlinear programming is one use of this material. There are others.

Chapter 17. Convex Sets

This chapter begins with concepts that are fundamental to analysis and to constrained optimization – the dot product of two vectors, the norm of a vector, the angle between two vectors, neighborhoods, open sets, closed sets, convex sets, and continuous functions. Two of the key results in this chapter are the "extreme value theorem" and the "supporting hyperplane theorem."

Chapter 18. Differentiation

This chapter is focused on the derivative of a function of two or more variables. A differentiable function is shown to be "well-approximated" by a plane or, in higher dimensions, by a hyperplane. The gradient of a differentiable function is introduced and is shown to point in the "uphill" direction, if it is not zero.

Chapter 19. Convex Functions

In this chapter, convex functions are defined, and ways in which to recognize a convex function are described. A key result in this chapter is that a convex function has a supporting hyperplane at each point on the interior of its domain.

Chapter 20. Nonlinear Programs

A set of optimality conditions for a linear program are re-interpreted as the "Karush/Kuhn/Tucker" conditions (or KKT conditions) for a nonlinear program. For a nonlinear program satisfies a particular "constraint qualification," a feasible solution is shown to be a global optimum if and only if it satisfies the KKT conditions. Weaker constraint qualifications are shown to lead to weaker results. This chapter includes a sketch of the Generalized Reduced Gradient (or GRG) method, which is used by Solver and by Premium Solver to find solutions to nonlinear programs.