

# Part IV–LP Theory

Part IV presents a more penetrating account of linear programming than is in earlier chapters. The theory in Part IV is important on its own, as are the algorithms to which it leads. They also prepare you for Part V (game theory) and for Part VI (nonlinear systems).

## **Chapter 10. Vector Spaces and Linear Programs**

Chapter 10 contains the information about vector spaces that relates directly to linear programs. You may find that much of this information is familiar, and you may find that linear programming strengthens your grasp of it.

## **Chapter 11. Multipliers and the Simplex Method**

As was noted in earlier chapters, shadow prices may not exist. The multipliers always exist. They may not be unique. Even when the multipliers are ambiguous, they are shown to account properly for the relative opportunity cost of each decision variable. The multipliers are also shown to guide the simplex method as it pivots.

## **Chapter 12. Duality**

In this chapter, the simplex method with multipliers is used to prove the “Duality Theorem” of linear programming. This theorem shows how each linear program is paired with another. Several uses of the Duality Theorem are presented in this chapter, and other uses appear in later chapters.

### **Chapter 13. The Dual Simplex Pivot and Its Uses**

This chapter introduces you to the “dual” simplex pivot, and it presents several algorithms that employ simplex pivots and dual simplex pivots. One of these algorithms is a one-phase “homotopy” that pivots from an arbitrary basis to an optimal basis. Another algorithm solves integer programs.