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Linear Programming and Generalizations

A Problem-based Introduction with
Spreadsheets

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Preface

The title of this book adheres to a well-established tradition, but “linear programming and generalizations” might be less descriptive than “models of constrained optimization.” This book surveys models that optimize something, subject to constraints. The simplest such models are linear, and the ideas used to analyze linear models generalize easily.

Over the past half century, dozens of excellent books have appeared on this subject. Why another? This book fuses five components:

- It uses examples to introduce general ideas.
- It engages the student in spreadsheet computation.
- It surveys the uses of constrained optimization.
- It presents the mathematics that relates to constrained optimization.
- It links the subject to economic reasoning.

Each of these components can be found in other books. Their fusion makes constrained optimization more accessible and more valuable. It stimulates the student’s interest, it quickens the learning process, it helps students to achieve mastery, and it prepares them to make effective use of the material.

A well-designed example provides context. It can illustrate the applicability of the model. It can reveal a concept that holds in general. It can introduce the notation that will be needed for a more general discussion.

Examples mesh naturally with spreadsheet computation. To compute on a spreadsheet is to learn interactively – the spreadsheet gives instant feedback. Spreadsheet computation also takes advantage of the revolution that has occurred in computer hardware and software. Decades ago, constrained optimization required specialized knowledge and access to huge computers. It was a subject for experts. That is no longer the case. Constrained optimization

has become vastly easier to learn and to use. Spreadsheets help the student to become facile with the subject, and it helps them use it to shape their professional identities.

Constrained optimization draws upon several branches of mathematics. Linear programming builds upon linear algebra. Its generalizations draw upon analysis, differential calculus, and convexity. Including the relevant math in a course on constrained optimization helps the student to master the math and to use it effectively.

Nearly every facet of constrained optimization has a close link to economic reasoning. I cite two examples, among many: A central theme of economics is the efficient allocation of scarce resources, and *the* canonical model for allocating scarce resources is the linear program. Marginal analysis is a key concept in economics, and it is exactly what the simplex method accomplishes. Emphasizing the links between constrained optimization and economics makes both subjects more comprehensible, and more germane.

The scope of this book reflects its components. Spreadsheet computation is used throughout as a teaching-and-learning aide. Uses of constrained optimization are surveyed. The theory is dovetailed with the relevant mathematics. The links to economics are emphasized.

The book is designed for use in courses that focus on the applications of constrained optimization, in courses that emphasize the theory, and in courses that link the subject to economics. A “user’s guide” is provided; it takes the form of a brief preview of each of the six Parts that comprise this book.

Acknowledgement

This book's style and content have been shaped by decades of interaction with Yale students. Their insights, reactions and critiques have led me toward a problem-based approach to teaching and writing. With enthusiasm, I acknowledge their contribution. This book also benefits from interactions with my colleagues on the faculty. I am deeply indebted to Uriel G. Rothblum, Kurt Anstreicher, Ludo Van der Heyden, Harvey M. Wagner, Arthur J. Swersey, Herbert E. Scarf and Donald J. Brown, whose influences are evident here.

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