

# Structurally Constrained Controllers

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Analysis and Synthesis

 Springer

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*...dedicated to*

**my mother** — *from* Somayeh Sojoudi

**my parents** — *from* Javad Lavaei

**my wife** — *from* Amir G. Aghdam

# Preface

Decentralized control of large-scale systems has been an active area of research in the past 40 years. The need for computationally efficient strategies to control high-order systems with a large number of inputs and outputs was the initial motivation for extensive research in this area. However, due to significant advances in computer technology, this may not be the main concern in practice. On the other hand, in more recent practical applications of large-scale control systems (such as cooperative control of autonomous vehicles), reliability is one of the most important design specifications; and of course reliability is a clear advantage of decentralized control structure over a centralized one because unlike a centralized structure, there is no single point of failure in a decentralized control configuration. Many universities in the world have a course entitled “large-scale control systems” in their graduate curriculum, where some classical results in decentralized control are covered. However, the need for a reference which addresses more recent problems in this area is clear. On one hand, the introduction of the new wave of applications of decentralized control in the past decade has provided a new dimension to this area of research. On the other hand, the rapid growth in computational power and resources has necessitated a major change in the composition of a systematic course in this area. The classical results in theory of decentralized large-scale systems build the foundation of the course contents, and the practical applications are important requirements of a meaningful engineering education, which makes the course more attractive from the student’s viewpoint. And these two components concurrently define effective teaching for a graduate course.

To the best of the authors’ knowledge, there is no book in the market which presents recent problems and methodologies in decentralized control of systems, without neglecting the important concepts introduced in earlier years, when the classical results in this area were developed. Therefore, professors often use a collection of relevant papers to address more recent research results and applications throughout the course. This book is an attempt to fill the gap between the classical results in decentralized control of large-scale interconnected systems developed in 1970’s and 80’s, and some important problems which were introduced by more recent applications of decentralized control, with a special focus on multi-agent systems appli-

cation. These problems include robust stability and stabilizability, simplest control structure required for stability, performance analysis, and information exchange between the control components. The authors hope that this can be a good reference book for any course in the area of large-scale control systems, and that the students will find it useful in identifying some open problems in this field of research.

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