NONLINEAR ANALYSIS AND VARIATIONAL PROBLEMS
Optimization has been expanding in all directions at an astonishing rate during the last few decades. New algorithmic and theoretical techniques have been developed, the diffusion into other disciplines has proceeded at a rapid pace, and our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in all areas of applied mathematics, engineering, medicine, economics and other sciences.

The series *Optimization and Its Applications* publishes undergraduate and graduate textbooks, monographs and state-of-the-art expository works that focus on algorithms for solving optimization problems and also study applications involving such problems. Some of the topics covered include nonlinear optimization (convex and nonconvex), network flow problems, stochastic optimization, optimal control, discrete optimization, multiobjective programming, description of software packages, approximation techniques and heuristic approaches.
NONLINEAR ANALYSIS AND VARIATIONAL PROBLEMS

In Honor of George Isac

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Springer
With our deepest appreciation, we dedicate this volume to the memory of our dearest friend and eminent mathematician, George Isac.
Preface

The papers published in this volume focus on some of the most recent developments in complementarity theory, variational principles, stability theory of functional equations, nonsmooth optimization, and various other important topics of nonlinear analysis and optimization.

This volume was initially planned to celebrate Professor George Isac’s 70th birthday by bringing together research scientists from mathematical domains which have long benefited from Isac’s active research passion. Unfortunately, George Isac passed away in February 2009 at the age of 69.

George Isac received his Ph.D. in 1973 from the Institute of Mathematics of the Romanian Academy of Sciences. He made outstanding contributions in several branches of pure and applied mathematics, including complementarity theory, variational inequalities, fixed point theory, scalar and vector optimization, theory of cones, eigenvalue problems, convex analysis, variational principles and regularization methods, as well as a number of other topics. In his long and outstanding career, he wrote more than 200 papers and 13 books. Professor Isac was an avid traveler who visited more than 70 universities around the globe and delivered approximately 180 research presentations. He also authored seven books on poetry. During his scientific career he collaborated with numerous mathematicians. His research papers contain very deep, original and beautiful results. Through his significant contributions, he earned a distinguished position and became an internationally renowned leading scholar in his research fields. Professor Isac’s prolific career was supported by the love and affection of his wife, Viorica. In fact her dedication, was so strong that she typed most of Isac’s manuscripts for his papers and books. We offer our sincerest sympathies to Viorica Isac on her monumental loss. Her husband was not only a wonderful mathematician but also a outstanding human being who will be greatly missed.

The submitted works of eminent research scientists from the international mathematical community are dedicated to the memory of this leading mathematician and very special colleague and friend, George Isac.
The contributions are organized into two parts. Part I focuses on selected topics in nonlinear analysis, in particular, stability issues for functional equations, and fixed point theorems.

In Chapter 1, Agratini and Andrica present a survey focusing on linear positive operators having the degree of exactness null and fixing the monomial of the second degree.

In their contribution, Amyari and Sadeghi present a Mazur–Ulam type theorem in non-Archimedean strictly convex 2-normed spaces and give some properties of mappings on non-Archimedean strictly 2-convex 2-normed spaces.

The emphasis of Cădariu and Radu is on extending some results of Isac and Rassias on ψ-additive mappings by giving a stability theorem for functions defined on generalized α-normed spaces and taking values in β-normed spaces.

The objective of Constantinescu’s contribution is on some investigates of \( W^\ast \)-tensor products of \( W^\ast \)-algebras.

In his contribution, Dragomir introduces a perturbed version of the median principle and presents its applications for various Riemann–Stieltjes integral and Lebesgue integral inequalities.

M. Eshaghi-Gordji et al. undertake the issues related to the stability of a mixed type additive, quadratic, cubic and quartic functional equation.

A short survey about the Hyers–Ulam stability of ψ-additive mappings is given by Găvruta and Găvruta in Chapter 7.

In their contribution, Jun and Kim investigate the generalized Hyers–Ulam stability problem for quadratic functional equations in several variables and obtain an asymptotic behavior of quadratic mappings on restricted domains.

The focus of Jung and Rassias is to apply the fixed point method for proving the Hyers–Ulam–Rassias stability of a logarithmic functional equation.

In their work, Park and Cui use the fixed point method to prove the generalized Hyers–Ulam stability of homomorphisms in \( C^\ast \)-algebras and Lie \( C^\ast \)-algebras and of derivations of \( C^\ast \)-algebras and Lie \( C^\ast \)-algebras for the 3-variable Cauchy functional equation.

In their paper, Park and Rassias use the fixed point method to prove the generalized Hyers–Ulam stability of certain functional equations in real Banach spaces.

The focus of Precup is on presenting new compression and expansion type critical point theorems in a conical shell of a Hilbert space identified with its dual.

The aim of Rus’s contribution is to give some Hyers–Ulam–Rassias stability results for Volterra and Fredholm integral equations by using some Gronwall lemmas.

In his contribution, Turinici presents a detailed study of Brezis–Browder’s principle. He shows that the version of Brezis–Browder’s principle for general separable sets is a logical equivalent of the Zorn–Bourbaki maximality result. In addition, several other interesting connections are established.

The second part of this volume discusses several important aspects of vector optimization and non-smooth optimization, as well as variational problems.

In Chapter 15, Balaj and O’Regan make use of the Kakutani–Fan–Glicksberg fixed point theorem to give an existence theorem for a generalized vector quasi-equilibrium problem.
In their contribution, Cojocaru et al. give a new method of tracking the dynamics of an equilibrium problem using an evolutionary variational inequalities and hybrid dynamical systems approach. They apply their approach to describe the time evolution of a differentiated product market model under incentive policies with a finite life span.

In Chapter 17, Daniele et al. give an overview of recent developments in the theory of generalized projections both in non-pivot Hilbert spaces and strictly convex and smooth Banach spaces. They also study the equivalence between solutions of variational inequalities and critical points of projected dynamical systems.

Eichfelder and Jahn’s aim is to present various foundations of a new field of research in optimization unifying semidefinite and copositive programming, called set-semidefinite optimization.

Giannessi and Khan extend the notion of image of a variational inequality by introducing the notion of an envelope for a variational inequality.

In Chapter 20, Goeleven develops a new approach to study a class of nonlinear generalized ordered complementarity problems.

Ha’s chapter presents a unified framework for the study of strong efficient solutions, weak efficient solutions, positive proper efficient solutions, Henig global proper efficient solutions, Henig proper efficient solutions, super-efficient solutions, Benson proper efficient solutions, Hartley proper efficient solutions, Hurwicz proper efficient solutions and Borwein proper efficient solutions of a set-valued optimization problem with or without constraints.

The contribution of Isac and Németh presents some mean value theorems for the scalar derivatives which are then used to develop a new method applicable to the study of the existence of nontrivial solutions of complementarity problems.

The chapter by Isac and Tammer presents new necessary conditions for approximate solutions of vector-valued optimization problems in general spaces by introducing an axiomatic approach for a scalarization scheme.

Lukkassen et al. undertake homogenization of sequences of integral functionals with natural growth conditions. Some means are analyzed and used to discuss some fairly new bounds for the homogenized integrand corresponding to integrands which are periodic in the spatial variable. Several applications are given.

In their contribution, Moldovan and Gowda employ duality and complementarity ideas and Z-transformations as well as discuss equivalent ways of describing the existence of common linear/quadratic Lyapunov functions for switched linear systems.

Motreanu’s focus is on the necessary conditions of optimality for general mathematical programming problems on a product space. Interesting applications to an optimal control problem governed by an elliptic differential inclusion are given.

In his contribution, Pascali’s focus is on studying variational inequalities with S-mappings.

In Chapter 28, a new completely generalized co-complementarity problem for fuzzy mappings is introduced. By using the definitions of $p$-relaxed accretive and $p$-strongly accretive mappings, the authors propose an iterative algorithm for computing the approximate solutions, and establish its convergence.
The contribution of Wolkowicz is aimed to illustrate how optimization can be used to derive known and new theoretical results about perturbations of matrices and sensitivity of eigenvalues.

It is our immense pleasure to express our utmost and deepest gratitude to all of the scientists who, by their works, participated in this tribute to honor Professor George Isac. We are grateful to the referees of the enclosed contributions. One of the editors (AAK) expresses his sincere gratitude to Prof. Sophia Maggelakis and Prof. Patricia Clark of RIT and Prof. G. Jailan Zalmai of NMU for their kindness and support.

Panos M. Pardalos
Themistocles M.Rassias
Akhtar A. Khan
May, 2009
Biographical Sketch of George Isac

George Isac was born on April 1, 1940, in Filipesti, Romania, a village in the district of Braila. His father was a village schoolteacher who fought in the First World War where he lost an arm. His mother was a housewife. George was the youngest of three children—he had an older sister and an older brother.

George enjoyed a happy childhood as evoked in his nostalgia-filled poems. His poetry has had much success in Romanian communities all over the world. It was said that on the first day of class, on the way to school, his father told him: I would like you to be the best student in your class. Young George took his father’s wish as an order and he strived with all his power to maintain this status during all of his
student years. He remained deeply attached not only to his birth village but also to
the village school. After the fall of the communist regime in Romania, he visited the
school several times and funded a scholarship to be granted every year to the best
student.

In 1955, George was admitted to the very prestigious college Nicolaie Balcescu
in Braila. The impressive number of great Romanian intellectuals who have gradu-
ated from that college is certainly due to the fact that the college had exceptionally
good teachers. One of them was the Romanian literature teacher who believed that
George was a real artistic gem and who was certain that George would follow a
higher education in the arts. However, George realized that under the communist
regime, it was too difficult for people to succeed in such a career without making
major moral concessions and he was not willing to make such concessions.

So it was, facing the prospect of producing a big disappointment to his favorite
teacher, he made the dramatic decision to enroll in the mathematics program at
the University of Bucharest. Still, he returned every year, full of emotion, to his
beloved college in Braila to long walks in Braila’s parks and along the banks of the
Danube River. It was there that he met Viorica Georgescu, who on May 8, 1965,
became his beloved wife and his inspiration for countless love poems that were
filled with enormous gratitude to her. Viorica gave him the most precious gift one
could receive: two wonderful children: Catalin and Roxana—the pride of the Isac
family.

At the University of Bucharest, George was a remarkable student and upon grad-
uation he was offered a position in the Department of Analysis, whose chairman at
that time was George Marinescu. There was an immediate and deep chemistry be-
tween the two Georges and they soon began to work together on a pioneering book
on analysis on ultra-metric fields (published in 1976). George Isac had wonderful
memories regarding his mentor George Marinescu and started to write a biography
about him that unfortunately remains unfinished.

George and a friend, Ion Ichim, were working part-time at the Institute of Math-
ematics, where I was a senior researcher nominated to direct doctoral studies. They
both came to my office one day and asked me to accept them as students in a Ph.D.
program. Knowing their value, I joyfully accepted them, as well as their subjects of
research: the area of functional analysis for George and the area of potential theory
for Ion.

In the meantime, in February 1972 I left Romania. At that time, the work on
their Ph.D. thesis was advanced but unfinished. Officially, Professor Marinescu was
nominated to take charge of directing their thesis, but due to his health problems,
my friend Aurel Cornea performed the real work. It was a pleasant job for him and
he offered them not only his help but also his friendship after graduation.

George Isac was offered a contract with University of Kinshasa in Zaire and
decided to accept it in order to escape to the West. Before his departure he paid a visit
to Aurel Cornea and told him about his intentions, demonstrating great confidence
in him, given the fact that the country was studded with secret police informers. He
mentioned that his intention was to go alone at first given the tough conditions there
and then bring his family after a while.
“Are you crazy?” asked Aurel. “Don’t you know what a source of blackmail a family left behind is for the secret police?” George insisted that his plan was sound, since he didn’t want to expose his family to hardship. Aurel walked thoughtfully through the room, then suddenly stopped and said: “This is what I have to say: Uncle George (a slightly ironical, yet kind address), take your family and go there. If it is too harsh, then put it on my account.”

George obeyed Aurel’s advice and fortunately had no regrets on his decision. He had an amazingly successful career in Canada, a country in which his family enjoyed every moment, and he was forever grateful to his friend Aurel for the advice to not leave them behind. Except for a short period of time at the University of Sherbrook, he always worked for the Royal Military College, first in Saint-Jean-sur-Richelieu, Quebec, and later in Kingston. He was also associated with Queen’s University, where he directed graduate student work.

While he was still in Romania, he started to pay attention to applied mathematics, a field in which he was able to use his functional analysis knowledge. He taught some courses in that field. In Canada, George evolved in his field and he achieved many accomplishments in applied mathematics, solving problems of complementarity, fixed point theorems with applications to decision theory, game theory, optimal control, Pareto problems, and nonlinear analysis, etc. Some basic concepts of these domains such as nuclear cones have been introduced by him.

George’s record of lifelong publications numbers over 200 papers and authorship or coauthorship of 13 books. There are around 640 quotations of his work in collaboration with 258 mathematicians. The world mathematics community sanctioned his mathematics contribution and he appeared as an invited speaker at countless international conferences and congresses. He was a member of the editorial committees for many mathematics periodicals. He received awards of excellence in mathematics, such as the “Spiru Haret” prize of the Romanian Academy of Science in 2003. He was also nominated for the title of Doctor Honoris Causa of the University Babeș-Bolyai, Cluj-Napoca, Romania.

One’s cannot talk about George Isac without saying something related to his poems, which were an important component of his personality. He started to write poetry only later in his life, probably because of the stress he experienced or because of an excessively busy mathematics schedule that did not allow him the peace of mind necessary for such an activity. However, he carried with him all of his life a kind of poetic archive, which overflowed tempestuously when the right time came, producing an impressive seven volumes of poems in just one decade, the eighth volume waiting for posthumous publication.

George’s poems are dominated by nostalgic memories of early childhood and adolescence. His birth village appears as the sacred place where forefathers’ traditions are still alive with colors and scents specific to every season, with flowers, birds, bugs, with rivers, cemeteries, wheat fields, vineyards, forest hills, and the usual childhood preoccupations such as flower picking in spring or tobogganing in winter. A lot of poems are dedicated to his parents’ house featuring an impressive garden and a very attentive mother. However, there are also dominant philosophical
problems, such as those related to life and death, treated mainly through the sieve of ancient oriental philosophy that he studied thoroughly.

As the religious man he was, George tackled the problem of life after death, trying to use poetic metaphors in order to revive in us the shiver of the absolute truth. There is also advice to not give too much importance to the superficial aspects of life, but rather concentrate on those that are deep and essential. He makes an acid indictment of our modern society, which shows signs of moral decadence.

George’s dreadful sickness surprised him while he was in full stride, which made it even worse. He had all kinds of mathematics projects in mind or under way: a new book, the biography of George Marinescu, and a book of personal recollections related to the communist era in Romania, including a recollection of his father’s tragic experience. It was not meant to be, and I regret that these marvelous projects are now forever lost.

Through his mathematics research, through his poems and through his teaching, George Isac brought a lot of light into this world. He lived life fully, offering us a rich harvest, similar to that of his birthplace fields described in his poems. Now, a new name must be added to the long list of science or arts personalities on the crown of Nicolae Balcescu College: that of George Isac, renowned personality in science and arts.

Benglen, April 2009

Corneliu Constantinescu
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