IUTAM Symposium on Relations of Shell, Plate, Beam, and 3D Models
Aims and Scope of the Series

The IUTAM Bookseries publishes the proceedings of IUTAM symposia under the auspices of the IUTAM Board.

For other titles published in this series, go to www.springer.com/series/7695
Ilia Vekua
(April 23, 1907 – December 2, 1977)
During its 2004 meeting in Warsaw the General Assembly of the International Union of Theoretical and Applied Mechanics (IUTAM) decided to support a proposal of the Georgian National Committee to hold in Tbilisi (Georgia), on April 23–27, 2007, the IUTAM Symposium on the Relation of Shell, Plate, Beam, and 3D Models, dedicated to the Centenary of Ilia Vekua. The scientific organization was entrusted to an international committee consisting of Philippe G. Ciarlet (Hong Kong), the late Anatoly Gerasimovich Gorshkov (Russia), Jorn Hansen (Canada), George V. Jaiani (Georgia, Chairman), Reinhold Kienzler (Germany), Herbert A. Mang (Austria), Paolo Podio-Guidugli (Italy), and Gangan Prathap (India).

The main topics to be included in the scientific programme were chosen to be: hierarchical, refined mathematical and technical models of shells, plates, and beams; relation of 2D and 1D models to 3D linear, non-linear and physical models; junction problems. The main aim of the symposium was to thoroughly discuss the relations of shell, plate, and beam models to the 3D physical models. In particular, peculiarities of cusped shells, plates, and beams were to be emphasized and special attention paid to junction, multibody and fluid-elastic shell (plate, beam) interaction problems, and their applications. The expected contributions of the invited participants were anticipated to be theoretical, practical, and numerical in character.

According to these premises, all the lecturers were invited personally for their active interest in the field covered by the symposium. In all there were 50 participants from 15 countries. The programme included an Opening Lecture, which was an impressive eulogy of Ilia Vekua, presented by B. Bojarsky (Poland), his former PhD student; the Closing Lecture was given by P. Podio-Guidugli (Italy), and there were 23 30-min-lectures. The afternoon sessions were ended by general discussions (round tables). The Georgian National Science Foundation has established a “Best Scientific Paper Award” to be bestowed on three young participants to the IUTAM Symposium. According to the decision of the International Scientific Committee of IUTAM
Symposium, these awards went to Natalia Chinchaladze (Georgia), Lorenzo Freddi (Italy), and Rainer Schlebusch (Germany).

The local arrangements of the symposium were in the hands of a committee consisting of Gia Avalishvili (I.Vekua Institute of Applied Mathematics), Natalia Chinchaladze (I.Vekua Institute of Applied Mathematics, Secretary), David Gordeziani (I.Vekua Institute of Applied Mathematics), George Jaiani (I.Vekua Institute of Applied Mathematics, Chairman), Gela Kipiani (Georgian Technical University), Tengiz Meunargia (Iv. Javakhishvili Tbilisi State University), Nugzar Shavlakadze (A.Razmadze Mathematical Institute), Ilia Tavkhelidze (Iv. Javakhishvili Tbilisi State University), Tamaz Vashakmadze (Iv. Javakhishvili Tbilisi State University).

The working sessions of the symposium were held in lecture-halls at the I. Vekua Institute of Applied Mathematics of Iv. Javakhishvili Tbilisi State University. In the occasion of the opening session, the symposium was welcomed by G. Khubua, Rector of Iv. Javakhishvili Tbilisi State University, N. Jokhadze, Director of the Georgian National Science Foundation, and D. H. Van Campen, IUTAM Secretary-General. Excursions and an interesting ladies programme complemented the scientific activities. In addition, the participants visited the Pantheon, the Georgian national cemetery of statesmen, scientists, and writers, with graves of I. Vekua and N. Muskhelishvili, founder and first president of the Georgian Academy of Sciences and the USSR National Committee of Theoretical and Applied Mechanics.

The volume includes 18 peer-reviewed papers presented at the symposium. The editors are indebted to Springer-Verlag for their courteous and effective production of these Proceedings.

March, 2008

G. Jaiani (Tbilisi)
P. Podio-Guidugli (Rome)
Contents

An Asymptotic Method for Solving Three-Dimensional Boundary Value Problems of Statics and Dynamics of Thin Bodies
Lenser A. Aghalovyan .................................................. 1

Multiscale Assessment of Low-Temperature Performance of Flexible Pavements
E. Aigner, R. Lackner, M. Spiegl, M. Wistuba, R. Blah,, H. Mang .... 21

On the Different Possibilities to Derive Plate and Shell Theories
Holm Altenbach, Johannes Meenen ..................................... 37

The Determination of Linear Frequencies of Bending Vibrations of Ferromagnetic Shell by Exact Space Treatment Bagdoev A.G., Vardanyan A.V., and Vardanyan S.V. .................... 49

Stability of a Rectangular Plate Capable of Transverse Shear Deformations
Vagharshak M. Belubekyan ................................................ 59

On a Problem of Thermal Stresses in the Theory of Cosserat Elastic Shells with Voids
Mircea Bîrsan ............................................................. 67

Vibration of an Elastic Plate Under the Action of an Incompressible Fluid
Natalia Chinchaladze ..................................................... 77

Some Remarks on Anisotropic Singular Perturbation Problems
Michel Chipot ........................................................... 91
On the Variational Derivation of the Kinematics for Thin-Walled Closed Section Beams
Lorenzo Freddi, Antonino Morassi, Roberto Paroni .......................... 101

Variational Dimension Reduction in Nonlinear Elasticity: A Young Measure Approach
Lorenzo Freddi, Roberto Paroni .................................................. 111

Joint Vibrations of a Rectangular Shell and Gas in It
Elena Gavrilova ........................................................................... 123

On Physical and Mathematical Moments and the Setting of Boundary Conditions for Cusped Prismatic Shells and Beams
George Jaiani ............................................................................... 133

Material Conservation Laws Established Within a Consistent Plate Theorie
Reinhold Kienzler, Dipak K. Bose .................................................. 147

A Small-Parameter Method for I. Vekua’s Nonlinear and Nonshallow Shells
Tengiz Meunargia ....................................................................... 155

The Extension and Application of the Hierarchical Beam Theory to Piezoelectrically Actuated Beams
DCD Oguamanam, C McLean,, JS Hansen ........................................ 167

Validation of Classical Beam and Plate Models by Variational Convergence
Paolo Podio-Guidugli .................................................................. 177

On the Simulation of Textile Reinforced Concrete Layers by a Surface-Related Shell Formulation
Rainer Schlebusch, Bernd W. Zastrau ............................................. 189

The Contact Problems of the Mathematical Theory of Elasticity for Plates with an Elastic Inclusion
Nugzar Shavlakadze .................................................................... 199

On the Basic Systems of Equations of Continuum Mechanics and Some Mathematical Problems for Anisotropic Thin-Walled Structures
Tamaz Vashakmadze .................................................................. 207

List of Participants ...................................................................... 219
Ilia Vekua
(April 23, 1907 - December 2, 1977)

April 23, 2007 is the day of 100th birth anniversary of Ilia Vekua, an eminent scholar in mathematics and mechanics.

Ilia Vekua’s research works cover various fields of mathematics and mechanics. Many of them are devoted to the theory of partial differential equations, in which Ilia Vekua took a great interest. In the analytical theory of linear differential equations of elliptic type with two independent variables, an important part was played by formulas of general representation of solutions by means of analytic functions of one complex variable. These formulas made it possible to widen considerably the field of application of the methods of the classical theory of analytic functions of a complex variable. Based on these studies, Ilia Vekua developed new methods for solving boundary value problems, which enabled him to investigate a vast class of boundary value problems formulated in nonclassical sense. The method he proposed for reducing boundary value problems to singular integral equations is one of the most powerful means for studies in this field. Concerning the merits of I. Vekua in the theory of singular integral equations, one can read in a well-known monograph “Singular Integral Equations” by N. Muskhelishvili: “Under the influence of a number of results obtained by the participants of the seminar and mainly due to I. N. Vekua’s fine works, the range of the problems I wished to study has considerably changed and I can note with a great and quite comprehensible pleasure that the most part of this book content should be considered as a result of joint work of a group of young scientific collaborators from the Tbilisi Mathematical Institute of the Georgian Academy of Sciences with I. Vekua and me”. Special mention should be made of a general boundary value problem for elliptic equations, which Ilia Vekua formulated and studied most completely. The well known boundary value problems of Dirichlet, Neumann and Poincaré are particular cases of this problem. Ilia Vekua derived the formulas of integral representation of holomorphic functions, which in the mathematical literature are named after him, and used them as an important tool in investigating the problem. Ilia Vekua is one of the founders of the theory of generalized analytic functions.

Ilia Vekua worked out several versions of the mathematical theory of elastic shells. In general, it should be stressed that all his works in mathematics were aimed at applications to topical problems of mechanics.

In recognition of his many pioneering contributions, Ilia Vekua

• was elected a Corresponding member of the Georgian Academy of Sciences in 1944,
• was elected a Corresponding member of the USSR Academy of Sciences and an Academician of the Georgian Academy of Sciences in 1946,
• was elected an Academician of the USSR Academy of Sciences in 1958,
• was elected a Foreign member of German Academy of Sciences in 1968,

• was elected a Foreign member of the Academy of Natural Sciences “Leopoldina” (Halle) in 1969,
• was elected a Foreign member of the Academy of Sciences of Literature and Art (Sicilian Academy of Sciences) in 1976,
• received the USSR Stalin Prize of the second degree for his monograph “New methods of solution of elliptic equations” (in Russian, published in 1948) in 1950,
• received the USSR Lenin Prize for his monograph “Generalized analytic functions” (in Russian, published in 1959) in 1963,
• received (posthumously) the State Prize for his research work “Some general methods of constructing various versions of the shell theory” (Russian, published in 1982) in 1984.

Ilia Vekua was born on April 23, 1907, in Abkhazian village Shesheleti (West Georgia). After finishing a secondary school in the West Georgian town Zugdidi in 1925, he moved to Tbilisi, the capital of Georgia, where he studied at the Faculty for Physics and Mathematics of Tbilisi State University. He graduated with honors in 1930 and, on the recommendation of Academician Niko Muskhelishvili, left Tbilisi for Leningrad (now St. Petersburg) to continue his education there as a post-graduate student at the USSR Academy of Sciences. His initial research was conducted under the supervision of the well-known mathematician A. N. Krylov. In Leningrad, Ilia Vekua published his first papers on problems of torsion and bending of elastic bars. He also worked on the theory of propagation of electric waves in an infinite layer with parallel plane boundaries and obtained the results which subsequently formed the basis of his thesis for the Candidate of Science degree.

After finishing the post-graduate course in 1933, Ilia Vekua returned to Tbilisi to work at his alma mater. He wholly devoted himself to scientific, educational and organizational activities. Ilia Vekua became an active participant in the famous seminar run by Niko Muskhelishvili. He delivered lectures on mathematical physics, calculus of variations, differential and integral equations and was one of the founders of the Mathematical Institute of the Georgian Branch of the USSR Academy of Sciences (now A. Razmadze Mathematical Institute).

In 1951, Ilia Vekua moved to Moscow where he was officially invited for permanent residence and work. Together with his outstanding colleagues and friends M. A. Lavrent’ev, I. G. Petrovskii, and S. L. Sobolev, he directed the research seminars at V. A. Steklov Mathematical Institute and M. V. Lomonosov Moscow University.

Ilia Vekua was the founding Rector (1959–1964) of Novosibirsk University. When living in Siberia, Ilia Vekua simultaneously combined several duties: he headed the theoretical department at the Hydrodynamics Institute of the Siberian Branch of the USSR Academy of Sciences, held the mathematical physics chair of Novosibirsk University, and supervised the work of several scientific seminars.
After the USSR National Committee on Theoretical and Applied Mechanics was formed in 1956, Ilia Vekua became a permanent member. From 1963 he was also a member of the National Committee of Soviet Mathematicians.

At the end of 1964, Ilia Vekua returned to Tbilisi, where he was elected vice-president of the Georgian Academy of Sciences (1964–1965) and head of the mathematics chair at Tbilisi State University (1966-1972). On his initiative and under his guidance, the Department of Mechanics was organized (1964) at A. Razmadze Mathematical Institute of the Georgian Academy of Sciences, and the Problem Laboratory of Applied Mathematics was founded (1966) at Tbilisi State University and shortly reorganized as the Institute of Applied Mathematics (1968). The latter institute is named after Ilia Vekua, who was its founder and remained its director and scientific leader (1968–1977) till the last days of his life. From 1972 to 1977, Ilia Vekua served as the president of the Georgian Academy of Sciences.

Through the last years of his life, in spite of his grave illness, Ilia Vekua continued to pursue his scientific, teaching and organizational activities. His last monographs were published posthumously. In September 1976, on Ilia Vekua’s suggestion, the IUTAM’s General Assembly decided to organize the 3rd International Symposium on the Theory of Shells in Tbilisi, Georgia. Ilia Vekua was appointed chairman both of the international scientific committee and of the national organizing committee. Preparations for the symposium were underway when the whole scientific world was deeply saddened by the untimely demise of Ilia Vekua on December 2, 1977. Nevertheless, the symposium which the IUTAM Bureau decided to dedicate to the memory of Ilia Vekua, was held in Tbilisi in August 22–28, 1978.

In recognition of his special services to mechanics in the occasion of the centenary of his birth, the IUTAM Symposium on Relation of Shell, Plate, Beam, and 3D Models was dedicated to I. Vekua. The symposium was held in Tbilisi on 23–27 April, 2007. The works of well-known scientists presented at the symposium are collected in this issue.

G. Jaiani
Main Publications of Ilia Vekua

(i) monographs

(ii) papers


Main Publications of Ilia Vekua xvii


27. On solutions of equation $\Delta u + \lambda^2 u = 0$. (Georgian) Soobshch. Akad. Nauk Gruz. SSR, 3(1942), No. 4, 307–314.


44. Integration of equations of a spherical shell. (Russian) Prikl. Mat. Mekh. 9(1945), No. 5, 368–388.


87. Equations and systems of equations of elliptic type. (Russian) Proc. IV
89. Theory of thin and shallow elastic shells with variable thickness.
(Russian) Applications of the theory of functions in continuum mechan-
90. Theory of thin shallow shells of variable thickness. (Russian) Trudy Tbilis.
Anwendungen der Mathematik in Ingenieurwiss. mit Rahmenthema, An-
wendungen elektronischer Rechenanlagen im Bauwesen, Bd. 1(1967),
260–280.
92. On construction of an approximate solution of the equation of a shallow
93. On conformal invariant differential forms in shell theory. Functional the-
etrical methods in partial differential equations, 303–311, Acad. Press,
1968.
94. On one version of the consistent theory of elastic shells. IUTAM Symp.
(Copenhagen, 1967), 59–84, Springer-Verlag, Berlin-Heidelberg-New York,
1969.
95. On the integration of a system of equations of an elastic equilibrium of a
96. On the integration of equilibrium equations of a cylindrical shell. (Russian)
97. On one class of an irregular elliptic system of first order equations.
98. On one class of elliptic systems with singularities. Proc. Intern. Conf. on
99. On one method of solving of the basic biharmonic boundary value problem
and the Dirichlet problem. (Russian) Some problems of mathematics and
101. On one trend of construction of shell theory. (Russian) Mechanics in the
103. On a functional equation of the theory of minimal surfaces. (Russian)
104. On two ways of constructing a noncontradictory theory of elastic shells.
(Russian) Proc. I All-Union School on the Theory and Numerical Methods
of Calculation of Shells and Plates, (Gegechkori), 5–10, Metsniereba, Tbilisi, 1975.