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Non-Equilibrium Phase Transitions

Volume I: Absorbing Phase Transitions

 Springer

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Malte Henkel, Haye Hinrichsen, Sven Lübeck

Non-equilibrium phase transitions

Volume 1 – Absorbing phase transitions

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Preface

“The career structure and funding of the universities [...] currently strongly discourages academics and faculties from putting any investment into teaching – there are no career or financial rewards in it. This is a great pity, because [...] it is the need to engage in dialogue, and to make things logical and clear, that is the primary defence against obscurantism and abstraction.”

B. Ward-Perkins, *The fall of Rome*, Oxford (2005)

This is the first volume of a planned two-volume treatise on non-equilibrium phase transitions. While such a topic might sound rather special and academic, non-equilibrium critical phenomena occur in much wider contexts than their equilibrium counterparts, and without having to fine-tune thermodynamic variables to their ‘critical’ values in each case. As a matter of fact, most systems in Nature are out of equilibrium. Given that the theme of non-equilibrium phase transitions of second order is wide enough to amount essentially to a treatment of almost all theoretical aspects of non-equilibrium many-body physics, a selection of topics is required to keep such a project within a manageable length. Therefore, Vol. 1 discusses a particular kind of non-equilibrium phase transitions, namely those between an active, fluctuating state and absorbing states. Volume 2 (to be written by one of us (MH) with M. Pleimling) will be devoted to ageing phenomena. The book is intended for readers who are familiar with general principles of statistical mechanics, at the level of a standard university course, and who have had some previous exposure to equilibrium critical phenomena and the renormalisation group. We aim at a presentation as self-contained and as accessible for the non-expert as possible.

This work was conceived and begun by two of us (MH & SL) in the summer of 2005. A little later, during the participation of two of us (MH & HH) in the workshop *Principles of Dynamics of Non-Equilibrium Systems* at the Newton Institute in Cambridge in spring 2006, the formal decision to write was made and a large part of the work was produced. We thank the Newton Institute and the organisers M. Evans, C. Godrèche, S. Franz and D.

Mukamel for the stimulating atmosphere which provided substantial encouragement with this project. Later, MH enjoyed the warm hospitality of and thanks cordially the Dipartimento di Fisica of the Università di Firenze and INFN - Sezione di Firenze, of the Centro de Física Teórica e Computacional (CFTC) at the Complexo Interdisciplinar of the Universidade de Lisboa, of the Instituut voor Theoretische Fysica at the Katholieke Universiteit Leuven and the Department of Theoretical Physics at the University of Saarbrücken for their support, which permitted him to make progress.

Our views on non-equilibrium physics have been formed by many friends and colleagues, sometimes through joint authorship on a paper, sometimes through intensive discussions and sometimes by some piece of advice. We are grateful to all of them, whether or not their contributions can be gleaned from the pages of this volume. It is a pleasure to gratefully thank P. Alnot, F. Baumann, B. Berche, G. Bonhomme, A. Capelli, E. Carlon, C. Chatelain, S.R. Dahmen, D. Dhar, E. Domany, S.B. Dutta, T. Enß, G. Foltin, P. Fulde, A. Gambassi, F. Ginelli, C. Godrèche, P. Grassberger, F. Hucht, W. Janke, H.-K. Janssen, D. Karevski, W. Kinzel, J.K. Krüger, R. Livi, J.-M. Luck, M. Lücke, C. Maes, S.S. Manna, J.F.F. Mendes, R.J. Meyer, G. Ódor, H. Park, M. Paeßens, I. Peschel, A. Picone, I.R. Pimentel, M. Pleimling, A. Politi, V.B. Priezhev, J. Ramasco, J. Richert, V. Rittenberg, R. Sanctuary, M.A.P. Santos, W. Selke, C.A. da Silva Santos, U. Schollwöck, R. Schott, G.M. Schütz, S. Stoimenov, U.C. Täuber, L. Turban, E. Vincent, D.E. Wolf, K.D. Usadel, J. Unterberger, C. Wagner, R.D. Willmann and J.-B. Zuber.

As everyone knows who is trying to compile a list of works from the literature, the exploding quantity of publications makes the production of a complete bibliography a task beyond human capabilities. The references we included are those which we needed in writing this volume and we sincerely apologise to any authors whose important contribution we might not have taken into account or might have covered inadequately.

We thank F. Hucht and the Institute of Theoretical Physics of the University of Duisburg for friendly support. The project has been overseen with diligence and patience by T. Spicer and we thank him and C. Caron for their help in bringing the first part of this work to completion.

Nancy, Würzburg, Gerlingen,
April 2008

Malte Henkel
Haye Hinrichsen
Sven Lübeck

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