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Ultracold Quantum Fields

Springer

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Preface

On June 19th 1999, the European Ministers of Education signed the Bologna Declaration, with which they agreed that the European university education should be uniformized throughout Europe and based on the two-cycle bachelor-master's system. The Institute for Theoretical Physics at Utrecht University quickly responded to this new challenge and created an international master's programme in Theoretical Physics which started running in the summer of 2000. At present, the master's programme is a so-called prestige master at Utrecht University, and it aims at training motivated students to become sophisticated researchers in theoretical physics. The programme is built on the philosophy that modern theoretical physics is guided by universal principles that can be applied to any subfield of physics. As a result, the basis of the master's programme consists of the obligatory courses Statistical Field Theory and Quantum Field Theory. These focus in particular on the general concepts of quantum field theory, rather than on the wide variety of possible applications. These applications are left to optional courses that build upon the firm conceptual basis given in the obligatory courses. The subjects of these optional courses include, for instance, Strongly-Correlated Electrons, Spintronics, Bose-Einstein Condensation, The Standard Model, Cosmology, and String Theory. The master's programme in Theoretical Physics is preceded by a summer school that is organized in the last two weeks of August to help prospective students prepare for the intensive master's courses. Short courses are offered in quantum mechanics, electrodynamics, statistical physics and computational methods, and are aimed at overcoming possible deficiencies in any of these subjects.

The idea of writing this book came about during the period of 2000-2005, when one of us was teaching the course on Statistical Field Theory for the abovementioned master's programme in Theoretical Physics. The lecture notes used for this course were an extended version of the lecture notes for the Les Houches summer school on *Coherent Atomic Matter Waves* that took place in 1999. Although these lecture notes, in combination with the lectures and tutorials, were supposed to be self-contained, in practice students often expressed a desire for more calculational details, applications and background material.

It was also during this period that the research field of ultracold atomic gases, pushed in particular by the impressive experimental progress since the first observation of Bose-Einstein condensation in 1995, made rapid developments that helped shape the field as we know it today. Nowadays, many experimental groups around the world can routinely prepare quantum degenerate gases of bosons, fermions, and various mixtures thereof. Moreover, the microscopic details of these atomic gases are well known and can be controlled very accurately, leading to the exciting possibility of addressing fundamental questions about interacting quantum systems in unprecedented detail. Because of this, it is also possible to perform ab initio theoretical calculations that allow for a quantitative comparison with experiments, such that the connection between theory and experiment is particularly close in this field of physics. There are various ways to perform these calculations, but most research topics can be dealt with in a unified manner by using quantum field theoretical methods. Although there are several textbooks available on quantum field theory, to date there does not exist a textbook that applies advanced quantum field theory, and in particular its functional formulation, to ultracold atomic quantum gases.

The level of this textbook is geared to students beginning with their master's and to graduate students already working in the field of ultracold atoms. To overcome the differences in educational background between the various students, the book has been divided into three parts which can in principle be read independently of each other. The first part briefly introduces elementary concepts from mathematics, statistical physics, and quantum mechanics which are indispensable for a full understanding of the rest of the book. Various important concepts that return later in the language of quantum field theory are introduced here in a more familiar setting. At the end of each chapter, there are references to various excellent textbooks that provide more background on each of the discussed topics. This part of the book is particularly aimed at the Utrecht Summer School in Theoretical Physics and provides the participants with the appropriate background material for the obligatory field theory courses that form the basis of the master's programme in Theoretical Physics. The second part of the book is devoted to laying the conceptual basis of the functional formulation of quantum field theory from a condensed-matter point of view. This part forms the core of the above mentioned Statistical Field Theory course, in which also the canonical topics of superfluidity and superconductivity of interacting Bose and Fermi gases are treated. The third part of the book is then largely aimed at applications of the developed theoretical techniques to various aspects of ultracold quantum gases that are currently being explored, such that the chosen topics give an idea of the present status of the field. It is our hope that, after having read this part, students will be well prepared to enter this exciting field of physics and be able to start contributing themselves to the rapid developments that are taking place today.

The knowledge presented in this book has been acquired through many collaborations and interactions with our colleagues over the last two decades. Here, we would like to sincerely thank everybody involved for that. It is unfortunately impossible to give everybody the proper credit for their contribution. As a result, both in this short word of thanks, as well as in citing references throughout the book, subjective choices are made and important contributions left out. Our main aim in citing has been to provide students with interesting additional reading material, and not to give an exhaustive overview of the enormous amount of literature in the field of ultracold atoms. We hope to be forgiven for that. With this in mind, we thank the following persons together with the members of their groups, namely Immanuel Bloch, Georg Bruun, Keith Burnett, Eric Cornell, Peter Denteneer, Steve Girvin, Randy Hulet, Allan MacDonald, Cristiane Morais Smith, Guthrie Partridge, Chris Pethick, Subir Sachdev, Cass Sackett, Jörg Schmiedmayer, Kevin Strecker, Peter van der Straten, Stefan Vandoren, and Eugene Zaremba for the collaborations that have led to joint publications. We also thank the postdoctoral researchers Usama Al Khawaja, Jens Andersen, Behnam Farid, Masud Haque, Jani Martikainen, Pietro Massignan, and Nick Proukakis, and the graduate students Michel Bijlsma, Marianne Houbiers, Michiel Bijlsma, Rembert Duine, Dries van Oosten, Gianmaria Falco, Lih-King Lim, Mathijs Romans, Michiel Snoek, Arnaud Koetsier, and Jeroen Diederix of the Utrecht Quantum Fluids and Solids Group. In particular, we mention Usama Al Khawaja, Rembert Duine, Dries van Oosten, and Nick Proukakis for their direct contributions to the recent applications that are discussed in the third part of the book. We also thank our experimental colleagues Immanuel Bloch, Eric Cornell, Randy Hulet, Wolfgang Ketterle, and Wenhui Li, for kindly providing us with the experimental data that has allowed us to compare the theory to experiment in this book. We thank Rembert Duine for providing several exercises and for many helpful comments on the manuscript. Furthermore we express our gratitude to Tom Spicer from Canopus Publishing for all his effort in bringing forth this book. We are especially grateful to Randy Hulet for more than 15 years of friendship and fruitful collaboration, from which we benefitted greatly, both personally and professionally.

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Utrecht, May 2008

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Contents