
DYNAMICAL THEORY OF DENDRITIC GROWTH IN CONVECTIVE FLOW

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DYNAMICAL THEORY OF DENDRITIC GROWTH IN CONVECTIVE FLOW

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Preface

The first draft of this monograph was written as a set of notes for the series of the lectures on the *Interfacial Wave Theory* of dendritic growth that I delivered in the State Key Laboratory of Crystal Growth at Shandong University, China, from August to October 2001. The targeted audience included the teachers, researchers and graduate students who were interested in the interdisciplinary areas of dynamics of pattern formation, material science, condensed matter physics and applied mathematics. A large portion of the material was later published as an invited review article in *Annual Review of Applied Mathematics and Mechanics* (2002). In writing this monograph I had an opportunity to further refine my previous works and correct some errors in the mathematical details found after their publications. The major revision of the monograph was done in the summer of 2002 during my visit to the State Key Laboratory for Studies of Turbulence and Complex Systems, the Department of Mechanics and Engineering Science at Peking University, where I completed my undergraduate study. I love the school then and even more so now for her ancient oriental academic tradition and the campus for her environmental quietness and natural beauty.

The Interfacial Wave (IFW) theory was first systematically described in my previous monograph “Interfacial Wave Theory of Pattern Formation: Selection of Dendritic Growth and Viscous Fingering in Hele–Shaw Flow” (Springer–Verlag, 1997). In that book, the objects of investigation were a variety of dynamic systems not restricted to dendritic growth. My attention was directed to discussing the common issues arising from various physical systems and exploring the common intrinsic mechanism underlying the phenomena.

Since publication of the monograph, the IFW theory for dendritic growth has further developed due to the extensive work done by my

students and me. Some new and profound problems have been analyzed. The present monograph attempts to summarize the new findings on dynamics of dendritic growth with convective flow.

We begin with a description of the macroscopic continuum approach for solidification problems and briefly review, with refined derivations, the IFW theory for the typical dendritic growth system without convection which provides readers with essential background for further study.

The main body of the book is devoted to a systematic study of the interactive dynamics of dendritic growth with convection flow in melt. In particular, it explores the effect of various types of convection flow on the selection and pattern formation of dendritic growth. These subjects have been of great interest to researchers in the broad fields of pattern formation, microgravity research and crystal growth.

This book will be useful for researchers, postdoctoral fellows and graduate students in the fields of condensed matter physics, materials science, microgravity science, theoretical and applied mechanics, chemical engineering, and applied mathematics.

I appreciate the *Spring Sun* Program launched by the Department of Education of China for the promotion of science and technology in some selected significant fields in China. My visit to Peking University was supported by this program. I thank Prof. Huang, Yong-Nian, Prof. Wei, Qing-Ding, and many staff members of the Laboratory for Studies of Turbulence and Complex Systems for their assistance and efforts which made my stay in the Swallow-Garden campus of Peking University most pleasant and productive.

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