

## Multi-Arm Cooperating Robots

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# Multi-Arm Cooperating Robots

*Dynamics and Control*

by

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# TABLE OF CONTENTS

<b>LIST OF FIGURES</b>	ix
<b>PREFACE</b>	xi
<b>1. INTRODUCTION TO COOPERATIVE MANIPULATION</b>	1
1.1 Cooperative Systems – Manipulation Systems	1
1.2 Contact in the Cooperative Manipulation	4
1.3 The Nature of Contact	4
1.4 Introducing Coordinate Frames	7
1.5 General Convention on Symbols and Quantity Designations	16
1.6 Relation to Contact Tasks Involving One Manipulator	18
<b>2. PROBLEMS IN COOPERATIVE WORK</b>	19
2.1 Kinematic Uncertainty	19
2.1.1 Kinematic uncertainty due to manipulator redundancy	19
2.1.2 Kinematic uncertainty due to contact characteristics	21
2.2 Force Uncertainty	22
2.3 Summary of Uncertainty Problems in Cooperative Work	24
2.4 The Problem of Control	25
<b>3. INTRODUCTION TO MATHEMATICAL MODELING OF COOPERATIVE SYSTEMS</b>	27
3.1 Some Known Solutions to Cooperative Manipulation Models	28
3.2 A Method to Model Cooperative Manipulation	30
3.3 Illustration of the Correct Modeling Procedure	37

3.4 Simulation of the Motion of a Linear Cooperative System	51
3.5 Summary of the Problem of Mathematical Modeling	54
<b>4. MATHEMATICAL MODELS OF COOPERATIVE SYSTEMS</b>	<b>57</b>
4.1 Introductory Remarks	57
4.2 Setting Up the Problem of Mathematical Modeling of a Complex Cooperative System	65
4.3 Theoretical Bases of the Modeling of an Elastic System	66
4.4 Elastic System Deformations as a Function of Absolute Coordinates	74
4.5 Model of Elastic System Dynamics for the Immobile Unloaded State	82
4.6 Model of Elastic System Dynamics for a Mobile Unloaded State	86
4.7 Properties of the Potential Energy and Elasticity Force of the Elastic System	89
4.7.1 Properties of potential energy and elasticity force of the elastic system in the loaded state translation	91
4.7.2 Properties of potential energy and elasticity force of the elastic system during its rotation in the loaded state	94
4.8 Model of Manipulator Dynamics	100
4.9 Kinematic Relations	101
4.10 Model of Cooperative System Dynamics for the Immobile Unloaded State	102
4.11 Model of Cooperative System Dynamics for the Mobile Unloaded State	104
4.12 Forms of the Motion Equations of Cooperative System	106
4.13 Stationary and Equilibrium States of the Cooperative System	118
4.14 Example	123
<b>5. SYNTHESIS OF NOMINALS</b>	<b>137</b>
5.1 Introduction – Problem Definition	138
5.2 Elastic System Nominals	142
5.2.1 Nominal gripping of the elastic system	142
5.2.2 Nominal motion of the elastic system	153
5.3 Nominal Driving Torques	165
5.4 Algorithms to Calculate the Nominal Motion in Cooperative Manipulation	166
5.4.1 Algorithm to calculate the nominal motion in gripping for the conditions given for the manipulated object MC	167

5.4.2	Algorithm to calculate the nominal motion in gripping for the conditions of a selected contact point	168
5.4.3	Algorithm to calculate the nominal general motion for the conditions given for the manipulated object MC	171
5.4.4	Algorithm to calculate the nominal general motion for the conditions given for one contact point	173
5.4.5	Example of the algorithm for determining the nominal motion	176
<b>6.</b>	<b>COOPERATIVE SYSTEM CONTROL</b>	<b>189</b>
6.1	Introduction to the Problem of Cooperative System Control	189
6.2	Classification of Control Tasks	191
6.2.1	Basic assumptions	191
6.2.2	Classification of the tasks	202
6.3	Choice of Control Tasks in Cooperative Manipulation	207
6.4	Control Laws	212
6.4.1	Mathematical model	212
6.4.2	Illustration of the application of the input calculation method	213
6.4.3	Control laws for tracking the nominal trajectory of the manipulated object MC and nominal trajectories of contact points of the followers	216
6.4.4	Behavior of the non-controlled quantities in tracking the manipulated object MC and nominal trajectories of contact points of the followers	223
6.4.5	Control laws to track the nominal trajectory of the manipulated object MC and nominal contact forces of the followers	229
6.4.6	Behavior of the non-controlled quantities in tracking the trajectory of the manipulated object MC and nominal contact forces of the followers	234
6.5	Examples of Selected Control Laws	236
<b>7.</b>	<b>CONCLUSION: LOOKING BACK ON THE PRESENTED RESULTS</b>	<b>251</b>
7.1	An Overview of the Introductory Considerations	251
7.2	On Mathematical Modeling	252
7.3	Cooperative System Nominals	254
7.4	Cooperative System Control Laws	256

7.5 General Conclusions about the Study of Cooperative Manipulation	257
7.6 Possible Directions of Further Research	258
<b>APPENDIX A: ELASTIC SYSTEM MODEL FOR THE IMMOBILE UNLOADED STATE</b>	261
<b>APPENDIX B: ELASTIC SYSTEM MODEL FOR THE MOBILE UNLOADED STATE</b>	269
<b>REFERENCES</b>	277
<b>INDEX</b>	283

# LIST OF FIGURES

1	Cooperative manipulation system	3
2	Contact	6
3	Cooperative work of the fingers on an immobile object	8
4	Kinematic uncertainty due to contact	22
5	Cooperative work of two manipulators on the object	23
6	Reducing the cooperative system to a grid	31
7	Approximation of the cooperative system by a grid	32
8	Linear elastic system	37
9	Approximating a linear elastic system	44
10	Block diagram of the model of a cooperative system without force uncertainty	51
11	Results of simulation of a ‘linear’ elastic system	54
12	Elastic system	63
13	Displacements of the elastic system nodes – the notation system	66
14	Angular displacements of the elastic system	76
15	Displacements of the elastic system	78
16	Planar deformation of the elastic system	83
17	Rotation of the loaded elastic system	95
18	Block diagram of the cooperative system model	106
19	Elastic system of two springs	113
20	Initial position of the cooperative system	123
21a	Simulation results for $\tau_i^j = 0, i, j = 1, 2, 3$	127
21b	Simulation results for $\tau_i^j = 0, i, j = 1, 2, 3$	128
22a	Simulation results for $\tau_1^1 = 50$ [Nm] and $\tau_2^1 = -50$ [Nm]	129
22b	Simulation results for $\tau_1^1 = 50$ [Nm] and $\tau_2^1 = -50$ [Nm]	130
22c	Simulation results for $\tau_1^1 = 50$ [Nm] and $\tau_2^1 = -50$ [Nm]	131
22d	Simulation results for $\tau_1^1 = 50$ [Nm] and $\tau_2^1 = -50$ [Nm]	132
22e	Simulation results for $\tau_1^1 = 50$ [Nm] and $\tau_2^1 = -50$ [Nm]	133

22f	Simulation results for $\tau_1^1 = 50$ [Nm] and $\tau_2^1 = -50$ [Nm]	134
22g	Simulation results for $\tau_1^1 = 50$ [Nm] and $\tau_2^1 = -50$ [Nm]	135
23	Nominal trajectory of the object MC	143
24	Elastic deviations from the nominal trajectory	146
25	Nominal trajectory of a contact point	163
26	'Linear' cooperative system	177
27	Nominals for gripping a manipulated object	181
28	Nominal input to a closed-loop cooperative system for gripping	182
29	Simulation results for gripping (open-loop cooperative system)	183
30	Nominals for manipulated object general motion	184
31	Nominal input to a closed-loop cooperative system for general motion	185
32	Simulation results for motion (open-loop cooperative system)	186
33	Mapping from the domain of inputs to the domain of states	194
34	Mapping from the domain of states to the domain of inputs	195
35	Mapping from the domain of inputs to the domain of outputs	195
36	Mapping from the domain of outputs to the domain of inputs	196
37	Mapping through the domain of states	196
38	Mapping of the control system domain	197
39	Structure of the control system	200
40	Mapping of the control object domain	201
41	Mapping of the cooperative manipulation domain	205
42	Global structure of the closed loop system	215
43	Motion in the plane of the loaded elastic system	224
44	Block diagram of the closed-loop cooperative system	240
45a	Gripping – tracking $Y_2^0$ and $Y_3^0$	241
45b	Gripping – tracking $Y_2^0$ and $Y_3^0$	242
46a	Gripping – tracking $Y_2^0$ and $F_{c2}^0$	243
46b	Gripping – tracking $Y_2^0$ and $F_{c2}^0$	244
47a	General motion – tracking $Y_2^0$ and $Y_3^0$	245
47b	General motion – tracking $Y_2^0$ and $Y_3^0$	246
48a	General motion – tracking $Y_2^0$ and $F_{c2}^0$	247
48b	General motion – tracking $Y_2^0$ and $F_{c2}^0$	248

# PREFACE

Under the notion ‘cooperative work’, is understood, in a widest sense the realization of a coordinated action of several participants (cooperators) engaged in a given task. Cooperative work is performed by a cooperative system consisting of cooperators and work object.

Cooperative work incorporates the joint work of the cooperators, their coordinated action in task execution, contact with the environment, and mutual contact of the cooperators, either directly or indirectly via the work object.

In joint work, the action of individual participants in the cooperation cannot be independent in time and space from the work (action) of the other participants. It is assumed that the actions of the cooperation participants take place simultaneously and not consecutively. Thereby cooperation means that each participant in the joint work carries out its own work taking care of the state of the other cooperation participants. Namely, to every different state of an individual cooperator corresponds an equal number of different states of the other cooperation participants. It is assumed that each cooperator obtains, in some way or other, information about the state of the other participants. The object on which cooperative work is performed, along with all cooperation participants, represent to an individual participant a dynamic environment with which it interacts.

There are a lot of tasks that can be performed in cooperation. Most often they are related to manipulating bulky objects whose weights exceeds the working capacities of the individual participants in the cooperation. For example, assembly of mechanical blocks carried out by several participants is a common case in technological practice. A frequent task is passing an object from one participant or group of participants in the cooperation to another participant or group of participants. In cooperative work, the participants perform mutually coordinated actions, while ensuring different types of contacts or avoiding them.

If, however, the extremities of an animal are considered as participants in cooperative work (manipulation or locomotion), then such synchronized motion is a specific cooperative task. The same also holds for the work (cooperation) of the fingers of a hand holding an object.

Analogous to the cooperative work of an animal's extremities is the robotic manipulation performed by several robots or by the fingers of an artificial hand. While object grasping and transferring, as well as the work on it, are the tasks of manipulation cooperation, synchronized work of the lower extremities represents locomotion cooperation that enables motion of the locomotion platform (vehicle) in the form of a bipedal or, more frequently, multipedal gait. Therefore, cooperative work of artificial systems has its biological counterpart in locomotion-manipulation activities of living beings. It can be said that results of studying active locomotion-manipulation mechanisms and their cooperation counterparts with living beings can be generally used in the corresponding procedures of the synthesis of artificial gait and control systems in manipulation and locomotion robotics.

When cooperative manipulation is concerned, a fundamental research task is to find out the appropriate way to control the system of robots and object in the work space at any stage of cooperative work. This requires an exact understanding of the physical nature of the cooperative system and deriving the mathematical basis for its description. In the realization of this goal, two crucial problems are encountered. The first of them is the occurrence of kinematic uncertainty and the second one is the force uncertainty in the mathematical description of the physical nature of the cooperative system. These problems have been considered by a number of authors [1–5, 12–20, 42–46, 50–55], and they can be interpreted simply as the impossibility to uniquely determine contact forces, driving torques of the manipulation mechanism, as well as kinematic quantities of cooperating robots, starting from the required motion of the object of cooperative manipulation.

On the basis of their research in the domain of cooperative manipulation, the authors of this monograph have recently come up with several consistent solutions concerning cooperative system control. This was achieved by solving three separate tasks that are essential for solving the problem of cooperative manipulation as a whole. The first task is related to understanding the physical nature of cooperative manipulation and finding a way for a sufficiently exact characterization of the cooperative system statics, kinematics, and dynamics. After successfully completing this task, in the frame of a second task, the problem of coordinated motion of the cooperative system is solved. Finally, as a solution to the third task, the control laws of cooperative manipulation are synthesized.

The starting point in dealing with the above three tasks of cooperative manipulation was the assumption that the problem of force uncertainty in cooperative manipulation can be solved by introducing elastic properties into the cooperative system. This monograph is concerned with the case when elastic properties are introduced only in that part of the cooperative system in which force uncertainty arises. Coordinated motion and control in cooperative manipulation are solved as the problem of coordinated motion and control of a mobile elastic structure, taking

into account the specific features of cooperative manipulation.

The contents of this monograph are organized into seven chapters.

Chapter 1 defines the notions and basic problems related to a cooperative system, cooperative manipulation and contact in cooperative manipulation. Also, coordinate systems used to describe the cooperative system characteristics are introduced.

In Chapter 2, some basic problems of cooperative manipulation are analyzed and a mathematical interpretation of the problem of kinematic uncertainty and force uncertainty is given.

Chapter 3 provides a concise systematization of previous solutions of the task of cooperative manipulation. It gives an analysis of the assumptions that are to be introduced in order to correctly solve the problem of cooperative manipulation under static conditions. It is shown that the problem cannot be solved without introducing the elastic properties of the loaded structure. Further, it is demonstrated that the cooperative system must be approximated by a mobile elastic structure. Also, it is shown how the problem of force uncertainty can be resolved by considering the deformation work of the elastic structure as a function of absolute coordinates. In other words, on the basis of such analysis, using a concrete simple example, a way is indicated for establishing a methodology of modeling dynamics of complex cooperative systems.

The difference between the way of considering statics and dynamics of the elastic structure of cooperative systems in the present book and in the available literature is in the following. In the literature [1–5], the authors start from the *a priori* implicit assumption that elastic displacements, needed to define the position of the elastic system in space, are not independent variables (state quantities), but they represent the displacements given in advance (like, for example, the known displacement of the support of an elastic structure when defining its statics [6, 7]). A consequence of such an *a priori* assumption is that the position of the unloaded state of elastic structure in the motion is known in advance, and the stiffness matrix of the elastic system is nonsingular. The elastic structure position in space can be defined by choosing any point, including a contact one. As a consequence, the manipulator internal coordinates that contact point belongs to, are given in advance, i.e. they are not state quantities. In deriving mathematical the model used in this work, it is assumed that all displacements of the elastic system (i.e. position of contact points and manipulated object mass center) are independent variables (state quantities), necessary and sufficient for describing elastic-system dynamics [8]. A consequence of such an assumption is that the stiffness matrix of the elastic part of the cooperative system is singular, i.e. it has to also contain the modes of motion of the elastic structure as of a rigid body.

Chapter 4 is concerned with the task of cooperative manipulation of a rigid

object by an arbitrary number of rigid manipulators, a task that has been most often considered in the literature. The task was modified by introducing elastic interconnections between the object and manipulators. The problem of modeling cooperative manipulation is analyzed in detail. In order to make the cooperative system properties more comprehensible, assumptions are introduced by which the problem of modeling is significantly simplified. Namely, the cooperative system is divided into its rigid part (manipulators) and elastic part (object and elastic interconnections). Each part is modeled separately using Lagrange equations. The elastic system model is derived on the basis of the description of its deformation work as a function of internal forces defined in dependence of absolute coordinates (extended method of finite elements [9]). The cooperative system dynamics is modeled for the displacement with respect to the elastic system unloaded state. This means that the reference coordinate frame is attached to the unloaded state of the elastic system. The general motion of the cooperative system is described in terms of absolute (external) coordinates, and the mathematical forms of motion equations are generalized. Stationary and equilibrium states of the cooperative system are analyzed in detail. The results obtained by model testing for selected examples show the consistency of our approach to modeling cooperative manipulation.

The problem of the synthesis of cooperative system nominals is essentially made more complex by introducing the elastic properties of the cooperative system [10]. Solving this problem is the subject of Chapter 5, where the nominals are synthesized using the properties of cooperative manipulation, as well as the properties of macro and micro motions. The cooperative system motion, in which the object is firstly gripped and then transferred, whereby the manipulator's motion does not significantly disturb the gripping conditions (i.e. the geometric configuration realized at the end of the gripping phase is not significantly changed) is adopted as the system's coordinated motion. The coordinated motion of the cooperative system is synthesized in a two-stage procedure, in which contact loads of the elastic system are approximately determined. On the basis of the approximate values of contact forces or driving torques, adopted as nominals, procedures are proposed for the synthesis of the other nominal quantities of the overall cooperative system. The synthesis procedures are illustrated by a simple example.

The control of cooperative manipulation is analyzed in Chapter 6 for the model of cooperative manipulation dynamics with the problem of force uncertainty resolved. The analysis encompasses definitions and criteria of controllability and observability of linear systems from the point of view of mapping the domains of inputs, states, and outputs. It is shown that the conclusions about mapping of linear systems can be applied without any change onto the mapping of the domains of inputs, states and outputs of the nonlinear systems. This was the basis for de-

iving conclusions on the controllability and observability of cooperative systems. Results of this analysis are applied to perform mapping between two of any of the following sets: the set of internal coordinates, the set of external coordinates, the set of driving torques, the set of contact forces, and the set of elasticity forces. A systematization of the controlled outputs along with the typification of control tasks in cooperative manipulation is carried out. Two types of tasks are selected [11]. Control laws are proposed for the asymptotically stable tracking of the object nominal trajectory and nominal trajectories of contact points of the manipulators-followers, along with control laws for the asymptotically stable tracking of the object trajectory and nominal contact forces at the contact points of the followers. The analysis also encompasses the behavior of uncontrolled quantities. The choice of the control laws and behavior of the controlled cooperative system are illustrated with a simple example.

The concluding Chapter 7 provides a brief survey of the research results that have been achieved in studying cooperative manipulation, which is the subject of this monograph. The conclusions are grouped according to particular topics. Also, some possible directions of the future research are indicated.

A complete derivation of the elastic system dynamic model for its immobile and mobile states is given in Appendices A and B, respectively.

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