

PLATE AND PANEL STRUCTURES OF ISOTROPIC, COMPOSITE
AND PIEZOELECTRIC MATERIALS, INCLUDING SANDWICH
CONSTRUCTION

SOLID MECHANICS AND ITS APPLICATIONS

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For a list of related mechanics titles, see final pages.

Plate and Panel Structures of Isotropic, Composite and Piezoelectric Materials, Including Sandwich Construction

by

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This textbook is dedicated to my beautiful wife Midge, who through her encouragement and nurturing over these last two decades, has made the writing of this book possible.

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PREFACE

Plates and panels are primary structural components in many structures from space vehicles, aircraft, automobiles, buildings and homes, bridges decks, ships, and submarines. The ability to design, analyze, optimize and select the proper materials and architecture for plates and panels is a necessity for all structural designers and analysts, whether the adjective in front of the “engineer” on their degree reads aerospace, civil, materials or mechanical.

This text is broken into four parts. The first part deals with the behavior of isotropic plates. Most metals and pure polymeric materials used in structures are isotropic, hence this part covers plates and panels using metallic and polymeric materials.

The second part involves plates and panels of composite materials. Because these fiber reinforced matrix materials can be designed for the particular geometry and loading, they are very often anisotropic with the properties being functions of how the fibers are aligned, their volume fraction, and of course the fiber and matrix materials used. In general, plate and panel structures involving composite materials will weigh less than a plate or panel of metallic material with the same loads and boundary conditions, as well as being more corrosion resistant. Hence, modern structural engineers must be knowledgeable in the more complicated anisotropic material usage for composite plates and panels.

Sandwich plates and panels offer spectacular advantages over the monocoque constructions treated above. By having suitable face and core materials, isotropic or anisotropic, sandwich plates and panels subjected to bending loads can be 300 times as stiff in bending, with face stresses 1/30 of those using a monocoque construction of a thickness equal to the two faces of the sandwich. Thus, for only the additional weight of the light core material, the spectacular advantages of sandwich construction can be attained. In Part 3, the analyses, design and optimization of isotropic and anisotropic sandwich plates and panels are presented.

In Part 4, the use of piezoelectric materials in beams, plates and panels are treated. Piezoelectric materials are those that when an electrical voltage is applied, the effects are tensile, compressive or shear strains in the material. Conversely, with piezoelectric materials, when loads cause tensile, compressive or shear strains, an electrical voltage is generated. Thus, piezoelectric materials can be used as damage sensors, used to achieve a planned structural response due to an electrical signal, or to increase damping. Piezoelectric materials are often referred to as smart or intelligent materials. The means to describe this behavior and incorporate this behavior into beam, plate and panel construction is the theme of Part 4.

This book is intended for three purposes: as an undergraduate textbook for those students who have taken a mechanics of material course, as a graduate textbook, and as a reference for practicing engineers. It therefore provides the fundamentals of plate and panel behavior. It does not include all of the latest research information nor the complications associated with numerous complex structures – but those structures can be studied and analyzed better using the information provided herein.

Several hundred problems are given at the end of Chapters. Most if not all of these problems are homework and exam problems used by the author over several decades of teaching this material. Appreciation is expressed to Alejandro Rivera, who as the first student to take the course using this text, worked most of the problems at the end of the chapters. These solutions will be the basis of a solutions manual which will be available to professors using this text who contact me.

Special thanks is given to James T. Arters, Research Assistant, who has typed this entire manuscript including all of its many changes and enhancements. Finally, many thanks are given to Dr. Moti Leibowitz who reviewed and offered significant suggestions toward improving Chapter 18, 19 and 20.