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TRANSFER OF POLARIZED LIGHT IN PLANETARY ATMOSPHERES

Basic Concepts and Practical Methods

by

JOOP W. HOVENIER

*Astronomical Institute "Anton Pannekoek",
University of Amsterdam, Amsterdam, The Netherlands*

CORNELIS VAN DER MEE

*Dipartimento di Matematica e Informatica,
Università di Cagliari,
Cagliari, Italy*

HELMUT DOMKE

Potsdam, Germany



SPRINGER SCIENCE+BUSINESS MEDIA, B.V.

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 978-1-4020-2889-2 ISBN 978-1-4020-2856-4 (eBook)
DOI 10.1007/978-1-4020-2856-4

Cover:

Illustration of the mirror symmetry relation for polarized light. See also page 75.

Printed on acid-free paper

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Originally published by Kluwer Academic Publishers in 2004

Softcover reprint of the hardcover 1st edition 2004

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PREFACE

PURPOSE

Over the past several years we have seen a variety of people obtaining a growing interest in the polarization of light scattered by molecules and small liquid or solid particles in planetary atmospheres. Some people first enjoyed observing brightness and colour differences of the clear or clouded sky before starting to wonder whether polarization effects might also be discerned. Others are attracted by the great potential of polarization measurements – whether from Earth or from spacecraft – for obtaining information on the composition and physical nature of the atmospheres of the Earth and other planets orbiting about the Sun or other stars. In addition, it is realized more and more by atmospheric scientists that significant errors in intensities (radiances) may occur when polarization is ignored in observations or computations of scattered light. Finally, many theoreticians of different kinds (astronomers, oceanologists, meteorologists, physicists, mathematicians) who are familiar with the already not so simple subject of transfer of unpolarized radiation can hardly resist the challenge of giving polarization its proper place in radiative transfer problems.

The main purpose of this monograph is to expound in a systematic but concise way the principal elements of the theory of transfer of polarized light in planetary atmospheres. Multiple scattering is emphasized, since the existing books on this topic contain little on polarization. On selecting the material for this book personal preferences, as always, played a certain role. Yet we have at least tried to primarily make our choices on the basis of criteria such as simplicity, fruitfulness, lasting value, practical applicability and potential for extension to more complicated situations.

READERSHIP

This book is chiefly intended for students and scientists who are interested in light scattering by substances in planetary atmospheres or other media. The latter involve, for instance, interplanetary and interstellar media, comets, rings around planets, circumstellar regions, water bodies like oceans and lakes, blood and a variety of artificial suspensions of particles in air or a liquid investigated in the laboratory. We expect that many investigators in these fields will find useful material in this book

for their problems of today and ideas for tomorrow.

The readers are assumed to have at least some basic knowledge of (classical) physics and mathematics. Some additional mathematical support is given in appendices. It seems likely that many readers will like to use the book for self-study. To facilitate this, problems and their solutions have been incorporated. Sometimes they do not only serve as practicing examples but also contain valuable information that did not easily fit in the main text.

STRUCTURE

This book deals with basic concepts and practical methods. In Chapter 1 we bring some order in the bewildering amount of descriptions, definitions and sign conventions used for treating polarized light. Some fundamentals as well as recent developments regarding single scattering by small particles are briefly discussed in Chapter 2. The next chapter focuses on scattering in plane-parallel atmospheres. We hope these three chapters and the appendices to be useful for fairly general purposes. Chapters 4 and 5 are devoted to practical computational methods and show how problems involving multiple scattering of polarized light in plane-parallel atmospheres can be solved. Only fairly general methods which have actually yielded accurate numbers, and not merely equations, are considered in this part of the book. First, in Chapter 4 approaches to calculate each order of scattering separately, as well as their sum, are considered. Chapter 5 is devoted to the adding-doubling method, which has proved to be of great value for computing the internal and emergent radiation of plane-parallel atmospheres.

A number of mathematical foundations of the theory of polarized light transfer in planetary atmospheres are not considered in this book. We intend to do so in a sequel to this book.

RESTRICTIONS

To keep the book within reasonable limits a number of restrictions had to be made. We mention the following.

First of all, we restrict ourselves to independent scattering by molecules and small particles like aerosols and cloud particles in planetary atmospheres and hydrosols in water bodies like oceans and lakes. In this book independent scattering means that when a beam of light enters a small volume element filled with particles each particle scatters light (radiation) independently of the other particles. At each moment the particles can be considered to be randomly positioned but constantly moving in space.

Secondly, we only consider elastic scattering, i.e. without changes of the wavelength, and we do not consider time variations on a macroscopic scale.

In the third place, our treatment of polarized light transfer is based on the classical radiative transfer theory in which energy is supposed to be transported

in a medium across surface elements along so called pencils of rays, while small (differential) volume elements are considered to be the elementary scattering units. This has turned out to provide sufficiently accurate results for the interpretation of most photometric and polarimetric observational data in and near the optical part of the spectrum. The precise relationship between the classical radiative transfer theory and electromagnetic theory has been obscure for a long time [See e.g Mandel and Wolf, 1995, Sec. 5.7.4], but it was considerably clarified in recent years, in particular by M.I. Mishchenko (2002, 2003) and Mishchenko et al. (2004), who used methods of statistical electromagnetics to give a self-consistent microphysical derivation of the radiative transfer equation including polarization.

Fourth, we only consider atmospheres that locally can be considered to be plane-parallel, i.e., built up of horizontal layers of infinite extent so that the optical properties of the atmosphere can only vary in the vertical direction.

Fifth, a huge amount of literature exists on Rayleigh scattering, i.e., the intensity and state of polarization of radiation coming from electric dipoles induced by incident radiation in any type of small entities, such as molecules and particles with sizes small compared to the wavelength inside and outside the particles [See e.g. Chandrasekhar, 1950; Van de Hulst, 1980]. In this book fairly little attention is given to Rayleigh scattering; it is only considered as a very special case of a more general theory.

Sixth, in this book we have chosen to refrain from a detailed treatment of applications of the theory to specific problems of transfer of polarized light in planetary atmospheres. Instead, we refer to some relevant papers at appropriate places.

GENERAL REMARKS

On choosing concepts, units and symbols for this book an important consideration has been for us that we wished to bridge and extend existing literature on single and multiple light scattering in planetary atmospheres and in particular books on these subjects [See e.g. Chandrasekhar, 1950; Van de Hulst, 1980; Sobolev, 1972]. For reasons of clarity and easy reference we have – following an idea of Van de Hulst (1980) – arranged certain formulae in a “Display,” which is a collection of formulae in tabular form. An extensive list of references is provided at the end of this book. Still, no attempt was made to mention every publication related to the subject matter of this monograph. Instead, the emphasis was put on books, review papers and research papers directly related to the text. In particular, we have been sparing of references to publications on light scattering and radiative transfer in which polarization was ignored or only very special cases, like Rayleigh scattering, were considered. English translations of publications in other languages, whenever known to us, have been mentioned along with the reference to the original work. Naturally we have tried as much as possible to avoid typos and other errors, though we cannot completely exclude their existence. Generally, however, we have given enough information in this book to enable the reader to verify if a particular statement or equation is correct.

ACKNOWLEDGMENTS

Over many years we have greatly profited from incisive discussions with a host of colleagues and students. In particular we wish to express our gratitude to J.F. de Haan for his extensive help and keen interest during our long lasting relationship with him. We are indebted to M.I. Mishchenko for lots of stimulating discussions, continuous encouragement and many useful comments on earlier versions of the manuscript. Parts of the manuscript were studied by Roelof Tamboer and Michiel Min and we are grateful for their remarks. One of us (JWH) gratefully acknowledges the kind hospitality extended to him by James E. Hansen and Larry D. Travis during several working visits at the NASA Goddard Institute for Space Studies in New York. Another of us (CvdM) is greatly indebted to the former Astronomy Group of the Free University of Amsterdam for their hospitality and stimulating discussions during countless working visits and to Michael Mishchenko for his hospitality during two short visits to the NASA Goddard Institute for Space Studies in New York.

The research of one of the authors (CvdM) was supported in part by the Italian Ministry of Education, Universities, and Research (under COFIN grant No. 2002014121) and by INdAM-GNCS and INdAM-GNFM.

J.W. Hovenier
C. van der Mee
H. Domke

April 8, 2004