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POLARIZATION IN SPECTRAL LINES

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*To Nadine and Vanessa,
and to Gianna*

PREFACE

Quod si tam celebris est apud omnes gloria Adamantis, atque varia ista opum gaudia, gemmae unionesque, ad ostentationem tantum placent, ut digitis colloque circumferantur; non minori afficiendos speraverim gaudio eos, quibus curiositatis conscientia quam deliciarum est potior, novitate corporis alicujus, instar crystalli translucidi, quod ex Islandia nuper ad nos perlatum est; cujus tam mira est constitutio, ut haud sciam, num alias magis naturae apparuerit gratia.

Erasmus Bartholinus, Experimenta crystalli islandici disdiaclastici

Apart from a few objects of our immediate neighborhood (the solar system), all the information on the physical phenomena taking place in the Universe comes from the radiation that the astronomical objects send into space and that is finally collected on earth by telescopes or other instruments. Among the different kinds of radiation, electromagnetic waves have by far played the most important role in the history of Astronomy – probably, it is not unrealistic to say that more than 99% of our present knowledge of the Universe derives from the analysis of the electromagnetic radiation.

Such radiation contains three different kinds of information, encoded into as many physical characteristics typical of any oscillatory propagation phenomenon: the propagation direction, the frequency and amplitude of the oscillation, and the oscillation direction – or polarization.

The first one is the most direct and the easiest to measure: the human eye is itself a suitable instrument, though of limited accuracy. As time passed, positional astronomy became more and more accurate thanks to the invention of the optical telescope, the introduction of photographic and digital techniques and, more recently, the development of technologies for producing images of a given region at different wavelengths via radio, infrared, X-ray and γ -ray telescopes, often operating on board of spacecrafts. Our present knowledge of the morphology and dynamics of the Universe, and of the different objects of which it is composed (from planets to stars, from nebulae to globular clusters, from galaxies to Active Galactic Nuclei and to clusters of galaxies) is based on the huge number of such observations that have been accumulating during several centuries.

However, even if very accurate, the measurement of the propagation direction of the electromagnetic radiation is inadequate to study other fundamental aspects of the physical Universe such as the composition, structure, and evolution of the different objects. To this aim, a detailed analysis of the frequency (or wavelength) distribution of the energy carried by the electromagnetic radiation is required, which was made possible by the invention of the spectrograph. Only through the systematic use of spectroscopic methods it has been possible to obtain a direct comprehension of the physical mechanisms which govern the equilibrium of stars, their birth, evolution and death, and the complicated processes taking place in the interstellar medium and in the nuclei of galaxies. Spectroscopy, which is also

at the basis of the idea of an expanding Universe, has played such a key role in the comprehension of the physical Universe that a new name, astrophysics, was introduced in the scientific lexicon to denote the astronomical research based on this technique.

The third, and often neglected, characteristic of the electromagnetic radiation is polarization. The earliest studies on polarization were performed around 1670 by the Danish scientist Erasmus Bartholinus, who was strongly impressed by the properties of a newly discovered crystal, the Iceland spar ('... whose behavior is so surprising that, as far as I know, never the grace of nature appeared more clearly') and who immediately realized that those properties could prove useful to improve human knowledge.

Since polarization is mainly related to the geometrical aspects of the emission process (rather than to its energetics), and since polarization measurements are often difficult to perform because of the intrinsic weakness of the signals, the study of polarization found its place in the astronomical research with some difficulty. But eventually the prediction of Erasmus Bartholinus was fully confirmed: some of the most important astronomical discoveries of the 20th century were made thanks to polarimetry – or, more properly, spectropolarimetry. Suffice it to quote the discovery of magnetic fields in the sun, stars, and the interstellar medium.

The first application of spectropolarimetry to the astronomical research dates back to 1908 when, using a Nicol prism as a polarizer and a Fresnel rhomb as a quarter-wave plate, George Ellery Hale succeeded in taking two spectra of the same area of a sunspot in opposite directions of circular polarization. The comparison of the spectra showed the presence of the typical signature induced by a strong magnetic field, the Zeeman effect.

Since 1908, things have considerably evolved from the technological point of view. Spectropolarimetric observations of the solar spectrum have now attained a sensitivity level which goes beyond the most optimistic expectations of only two or three decades ago. The first dedicated instrument for the measurement of Stokes parameters profiles in Fraunhofer lines, the 'mythic' Stokes-I polarimeter, developed in the mid 1970s at the High Altitude Observatory, hardly attained a sensitivity of 1%. Nowadays, sensitivity in solar spectropolarimetry has reached the level of 10^{-5} for spatially unresolved observations, and approximately one order of magnitude less for observations at high spatial and temporal resolution. It has to be expected that these limits will be rapidly overcome by the next generation polarimeters and that the same technologies will be adapted to galactic and extragalactic observations.

The dramatic increase of polarimetric sensitivity in solar observations has raised a serious challenge to the theoretical interpretation. Polarization in spectral lines is indeed a subtle phenomenon since, in astrophysical plasmas, there are several physical mechanisms that can generate polarization signatures in line profiles and many others that can modify them during the propagation. Some of these mechanisms have been known for a long time from laboratory atomic physics. They are – just to mention the most remarkable – the Zeeman effect, resonance polarization, and the Hanle effect. Other mechanisms are characteristic of optically thick plasmas, and are related to the propagation of radiation in anisotropic media.

They are known under the general names of dichroism and anomalous dispersion, though in special cases different denominations are often used (inverse Zeeman effect, magneto-optical effects, Faraday rotation, Faraday pulsation, etc.).

These processes have mostly been studied in specific physical contexts, for different purposes and at different levels of sophistication, and the scientific literature on the subject is scattered across books and journals, spanning almost a century of active research. For this reason we felt that a book capable of describing, in a unique and self-consistent formalism, all the known physical phenomena that may affect the polarization signatures of spectral lines, might be welcome to the scientific community. The diagnostic content of spectropolarimetry is high, but the correct interpretation of the observations requires a full understanding of the physics underlying the generation and transfer of polarized radiation.

The redaction of this book required several years. We might try to say, like Huygens in the preface to his *Treatise on light*, ‘The reason is that I wrote it rather carelessly in the Language in which it appears, with the intention of translating it into Latin, so doing in order to obtain greater attention to the thing’,¹ but we feel it would be hardly believed. The true reason is that the theory of spectropolarimetry is complicated, because it implies some knowledge of several subjects: atomic physics, quantum mechanics (with special emphasis on the theory of angular momentum and of the density matrix), quantum electrodynamics, radiative transfer (both under LTE and Non-LTE conditions).

Moreover, spectropolarimetry is full of traps: among all the disciplines in astrophysics, there can hardly be found one where more attention has to be payed to each single definition, each transformation, each physical application. Sign errors are especially insidious, as most remarkably shown by the classical example of circular polarization in a given wing of a spectral line formed in a magnetic atmosphere. There are four operations which produce a sign change in such polarization, and obviously, there are as many possibilities to make a sign error. To have a sign switch, one can: a) invert the direction of the magnetic field; b) interchange the red with the blue wing; c) use the opposite definition of positive and negative circular polarization; d) consider an emission line instead of an absorption line. This is just an example, but it shows very well the subtleties of the subject. We tried to make the exposition as clear as possible by using everywhere the same definitions and conventions, and by carefully describing all the mathematical developments.

We hope that this book may be useful to the next generations of scholars that will like to enter the field of spectropolarimetry, solar and non-solar. And we hope that it may contribute to make this research field more accessible and less hermetic, thus attracting more and more scientists to the fascinating world of the Stokes parameters profiles and of their interpretation.

Arcetri, March 2004

*Egidio Landi Degl’Innocenti
Marco Landolfi*

¹ Christiaan Huygens, *Treatise on light* (1690), translated by S.P. Thompson, Dover Publications, New York, 1962.

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