

A COURSE IN
PURE AND APPLIED MATHEMATICS

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BY

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PREFACE

It is customary in the preface to a text book to write about the scope of the book, to express some general ideas of the authors relevant to the subject matter and to express their thanks. We will not break with tradition here, although we hope in the text which follows that we have not let ourselves be so bound.

The book is intended for students who have already taken a first course of calculus and revision material appropriate to such a course is included. We have attempted to present material logically but we have not aimed at being rigorous, so that, for example, proofs requiring the notion of continuity are not included. We have not followed the path of those interested only in applications of mathematics nor of those who are interested only in that part of mathematics which gives it a claim to be ranked among the arts and humanities. We believe that history points to the best being found in a blending of both aspects so that understanding and the acquisition of results go hand in hand.

The setting up of appropriate algebraic and differential equations has been assumed the central part of applied mathematics and a number of exercises have been included which have this as the end point. The introduction of literal data and the use of general methods rather than special procedures have been considered important.

The book could provide a two year course, the first of which could be sections 1·1–1·52, 2·1–2·7, 3·1–3·7, 4·1–4·64, 5·1–5·3, 6·1–6·32, 8·1–8·54, 9·1–9·5, 10·1–10·5, 12·1–12·4 and 13·1. Set 1 of the General Miscellaneous Exercises is limited to these sections.

We thank Dr H. Laszlo formerly of R.M.I.T., now of Monash University, for many valuable discussions and ideas, Dr K. M. Bing of R.M.I.T. for reading the manuscript and making many suggestions for its improvement, the staff of the Mathematics Department of R.M.I.T. for their cooperation in checking many answers, Mr J. C. Vickery for his painstaking care in drawing the diagrams and the Education Department of Victoria for permission to use questions from their examination papers.

H. J. H.
D. A. H.

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