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Singular Spectrum Analysis

Using R

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PREFACE

Time series analysis is crucial in the modern world as time series data emerge naturally in the field of statistics. As a result, the application of time series analysis covers diverse areas, including those relating to ecological and environmental data, medicine and more importantly economic and financial time series analysis. In the past, time series analysis was restricted by the necessity to meet certain assumptions, for example, normality. In addition, the presence of outlier events, such as the 2008 recession, which causes structural changes in time series data, has further implications by making the time series non-stationary. Whilst methods have been developed using condemning time series models, such as variations of autoregressive moving average models, ARIMA models, such methods are largely parametric. In contrast, Singular Spectrum Analysis (SSA) is a non-parametric technique and requires no prior statistical assumptions such as stationarity or linearity of the series and works with both linear and non linear data. In addition, SSA has outperformed methods such as ARIMA, ARAR and Holt-Winters in terms of forecast accuracy in a number of applications. The SSA method consists of two complementary stages, known as decomposition and reconstruction, and both stages include two separate steps. At the first stage the time series is decomposed and at the second stage the original series is reconstructed and this series, which is noise free, is then used to forecast new data points. The practical benefits of SSA have resulted in its wide using over the last decade. As a result, the successful applications of SSA can now be identified across varying disciplines such as physics, meteorology, oceanology, astronomy, medicine, climate data, image processing, physical sciences, economics and

finance. Practically there are few programs, such as SAS and Caterpillar, which allow performing the SSA technique, but these require payments which are sometimes not economical for an individual researcher. R is an open-source software package that was developed by Robert Gentleman and Ross Ihaka at the University of Auckland in 1999. Since then, it has experienced a huge growth in popularity within a short span of time. R is a programme which allows the user to create their own objects, functions and packages. The R system is command driven and it documents the analysis steps making it easy to reproduce or update the analysis and figure errors. R can be installed on any platform and is license free. A major advantage with R is that it allows integrating and interacting with other paid platforms such as SAS, Stata, SPSS and Minitab. Although there are some books in the market relating to SSA, this book is unique as it not only details the theoretical aspects underlying SSA, but also provides a comprehensive guide enabling the user to apply the theory in practice using the R software. This book provides the user with step-by-step coding and guidance for the practical application of the SSA technique to analyse their time series databases using R. We provided some basic R commands in Appendix, so the readers who are not familiar with this language please learn the very basics in the Appendix at first.

The help of Prof. Kerry Patterson and Prof. Michael Clements in editing the text is gratefully acknowledged. Discussions with Kerry and Michael helped to clarify various questions treated on the following pages. We thank both for their encouragement.

As this book endeavours to provide a concise introduction to SSA, as well as to its application procedures to time series analysis, it is mainly aimed at masters and Ph.D.'s students with a reasonably strong stats/math background who wants to learn SSA, and is already acquainted with R. It is also appropriate for practitioners wishing to revive their knowledge of times series analysis or to quickly learn about the main mechanisms of SSA. On the time series side, it is not necessary to be an expert on what is popularly called Box-Jenkins modelling. In fact this could be a disadvantage since SSA modelling start from a somewhat different point and in doing so challenges some of the underlying assumptions of the Box-Jenkins approach.

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June 2018

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