

Progress in Mathematics

Volume 287

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Higher Structures in Geometry and Physics

In Honor of Murray Gerstenhaber
and Jim Stasheff

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ISBN 978-0-8176-4734-6 e-ISBN 978-0-8176-4735-3
DOI 10.1007/978-0-8176-4735-3
Springer New York Dordrecht Heidelberg London

Mathematics Subject Classification (2010): 53D55, 16S80, 17B25, 53D17

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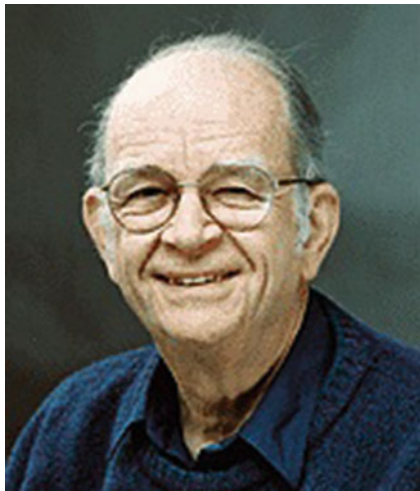
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Murray Gerstenhaber



Jim Stasheff

Foreword

You have tendered us a great honor.

Beyond that, we have listened throughout this conference with fascination and pleasure to the advances those invited here have made in the bloodless battle against an ever-retreating foe¹ in which we are all engaged. In this battle, where we start from ground gained by our teachers and predecessors – some of whom held forth in these same halls – our only weapons are those of thought. We devote our lives to this struggle against ignorance driven mainly by the hope of occasional triumphs and the joy of experiencing them when they occur.

Rewards and honors are usually bestowed on individuals but should really be shared by our whole community of mathematicians. Without each other's support, little progress is possible. And so while we thank you for the recognition you have given us and acknowledge our indebtedness to the organizers Professors Alberto Cattaneo and Ping Xu, to the institutions which provided the funds that enabled us to gather here the European Science Foundation, the National Science Foundation, and the National Security Agency of the USA, and to the Institut Henri Poincaré – for providing the venue in which we gathered, we also recognize our indebtedness to the continuing community of scholars, which we have been privileged to join. We have freely used their ideas. We hope that those newly joined in the battle will share our enthusiasm, go farther tomorrow than we, and look on our yesterday with appreciation that the ground we won has been not just for ourselves but for them.

Again, you have tendered us a great honor, but the celebration is really one of our communal spirit. Our knowledge will never be perfect nor will our understanding of the mysteries of mathematics ever be complete, but our searching, too, will never cease. And may we and those who succeed us gather often to celebrate this restless spirit.

¹ André Weil.

Preface

This book arose from a meeting centered on higher algebraic structures that are now ubiquitous in various areas of mathematics (algebra, algebraic topology, differential geometry, algebraic geometry, mathematical physics) and theoretical physics (quantum field theory, string theory). These structures provide a common language essential for the study of deformation quantization, theory of algebroids and groupoids, symplectic field theory, and much more.

These higher algebraic structures first appeared in 1963, in Murray Gerstenhaber's¹ *The cohomology structure of an associative ring* and in Jim Stasheff's *Homotopy associativity of H -spaces. I, II.*² In these fundamental publications, one finds the introduction of the notions that were to be called a Gerstenhaber algebra (developed in part to understand algebraic deformation theory) and an A_∞ algebra (developed in part to understand higher homotopies). While the relation between these notions was not immediately recognized, the ideas of higher homotopies and algebraic deformation would merge decades later and they are now permanently intertwined. The ideas of Gerstenhaber and Stasheff are present in every contribution of this volume.

¹ Ann. Math. **78**, 267–288 (1963).

² Trans. Am. Math. Soc. **108**, 275–292 (1963).

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