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An Introduction to Queueing Theory

Modeling and Analysis in Applications

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*In memory of my parents,
Vaidya P. Ishwar and Parvati Bhat*

Contents

Preface	xi
1 Introduction	1
1.1 Basic System Elements	1
1.2 Problems in a Queueing System	2
1.3 A Historical Perspective	4
1.4 Modeling Exercises	11
2 System Element Models	13
2.1 Probability Distributions as Models	13
2.1.1 Deterministic Distribution (D)	14
2.1.2 Exponential distribution; Poisson process (M)	14
2.2 Identification of Models	18
2.2.1 Collection of Data	18
2.2.2 Tests for Stationarity	18
2.2.3 Tests for Independence	19
2.2.4 Distribution Selection	19
2.3 Review Exercises	20
3 Basic Concepts in Stochastic Processes	23
3.1 Stochastic Process	23
3.2 Point, Regenerative, and Renewal Processes	23
3.3 Markov Process	24
4 Simple Markovian Queueing Systems	29
4.1 A General Birth-and-Death Queueing Model	29
4.2 The Queue $M/M/1$	34
4.2.1 Departure Process	40
4.3 The Queue $M/M/s$	43
4.4 The Finite Queue $M/M/s/K$	51
4.5 The Infinite-Server Queue $M/M/\infty$	58

4.6	Finite-Source Queues	59
4.7	Other Models	62
4.7.1	The $M/M/1/1$ System	62
4.7.2	Markovian Queues with Balking	64
4.7.3	Markovian Queues with Reneging	66
4.7.4	Phase-Type Machine Repair	66
4.8	Remarks	68
4.9	Exercises	68
5	Imbedded Markov Chain Models	75
5.1	Imbedded Markov Chains	75
5.2	The Queue $M/G/1$	76
5.3	The Queue $G/M/1$	98
5.4	Exercises	112
6	Extended Markov Models	115
6.1	The Bulk Queue $M^{(X)}/M/1$	115
6.2	The Bulk Queue $M/M^{(X)}/1$	118
6.3	The Queues $M/E_k/1$ and $E_k/M/1$	120
6.4	The Bulk Queues $M/G^K/1$ and $G^K/M/1$	123
6.5	The Queues $E_k/G/1$ and $G/E_k/1$	126
6.6	The Queue $M/D/s$	126
6.7	The Queue $M/M/1$ with Priority Disciplines	127
6.8	Exercises	138
7	Queueing Networks	141
7.1	Introduction	141
7.2	The Markovian Node Network	142
7.3	Queues in Series	144
7.4	Queues with Blocking	147
7.5	Open Jackson Networks	150
7.6	Closed Jackson Networks	152
7.7	Cyclic Queues	154
7.8	Operational Laws for Performance Analysis	155
7.9	Remarks	157
7.10	Exercises	158
8	Renewal Process Models	161
8.1	Renewal Process	161
8.2	Renewal Process Models for Queueing Systems	166
9	The General Queue $G/G/1$ and Approximations	169
9.1	The General Queue $G/G/1$	169
9.2	Little's Law $L = \lambda W$	173
9.3	Approximations	175
9.4	Diffusion Approximation	178

9.5	Fluid Approximation	180
9.6	Remarks	183
10	Statistical Inference for Queueing Models	185
10.1	Introduction	185
10.2	Birth-and-Death Process Models	187
10.3	Imbedded Markov Chain Models for $M/G/1$ and $G/M/1$	191
10.4	The Queue $G/G/1$	193
10.5	Other Methods of Estimation	194
10.6	Tests of Hypotheses	197
10.7	Control of Traffic Intensity in $M/G/1$ and $G/M/1$	197
10.8	Remarks	199
11	Decision Problems in Queueing Theory	201
11.1	Introduction	201
11.2	Performance Measures for Decision Making	202
11.3	Design Problems in Decision Making	202
11.4	Control Problems in Decision Making	205
12	Modeling and Analysis Using Computational Tools	207
12.1	Mean Value Analysis	207
12.2	The Convolution Algorithm	211
12.2.1	Computing Other Performance Measures	213
12.3	Simulation	214
12.4	MATLAB	217
12.5	Exercises	223

Appendices

A	Poisson Process: Properties and Related Distributions	229
A.1	Properties of the Poisson Process	229
A.2	Variants of the Poisson Process	231
A.3	Hyperexponential (HE) Distribution	233
A.4	Erlang Distribution (E_k)	234
A.5	Mixed Erlang Distributions	234
A.6	Coxian Distributions; Phase-Type Distribution	235
A.7	A General Distribution	236
A.8	Some Discrete Distributions	236
B	Markov Process	239
B.1	Kolmogorov Equations	239
B.2	The Poisson Process	240
B.3	Classification of States	242
B.4	Phase-Type Distributions	243

C	Results from Mathematics	247
C.1	Riemann–Stieltjes Integral	247
C.2	Laplace Transforms	248
C.3	Generating Functions	250
	References	253
	Index	265

Preface

There are several books on queueing theory available for students as well as researchers. At the low end of mathematical sophistication, some provide usable formulas in a recipe fashion. At the high end there are research monographs on specific topics and books with an emphasis on theoretical analysis. In between there are a few textbooks with one common feature: all of them require an adequate background knowledge of probability and Markov processes that can be acquired normally with a semester-length graduate course. Consequently, most people who deal with the modeling and analysis of queueing systems either do not take a course on the subject because it would require an extra semester, or take a course on queueing systems without the necessary background and learn only how to use the results. This book is addressed to remedy this situation by providing a one-semester foundational introduction to the theory necessary for modeling and analysis of systems while developing the essential Markov process concepts and techniques using queueing processes as examples.

Some of the key features of the book also distinguish it from others. Its introductory chapter includes a historical perspective on the growth of queueing theory in the last 100 years. With an emphasis on modeling and analysis it deals with topics such as identification of models, collection of data, and tests for stationarity and independence of observations. It provides a rigorous treatment of basic models commonly used in applications with references for advanced topics. It gives a comprehensive discussion of statistical inference techniques usable in the modeling of queueing systems and an introduction to decision problems in their management. The book also includes a chapter, written by computer scientists, on the use of computational tools and simulation in solving queueing theory problems.

The book can be used as a text for first-year graduate students in applied science areas such as computer science, operations research, and industrial and/or systems engineering, and allied fields such as manufacturing and communication engineering. It can also serve as a text for upper-level undergraduate students in mathematics, statistics, and engineering who have a reasonable background in calculus and basic probability theory. This book is the product of the author's experience in teaching

queueing theory for 40 years at various levels to students with or without the necessary background in stochastic processes.

The mathematical background assumed here is a two- or three-semester course in calculus, some exposure to transforms and matrices, and an introductory course in probability and statistics—all at the undergraduate level. An appendix on mathematical results provides some of the essential theorems for reference. Instructors may request a guide to the solutions of exercises via the Birkhäuser website at www.birkhauser.com/978-0-8176-4724-7.

The book does not advocate any specific software for the numerical analysis of queueing problems. The one chapter on modeling and analysis using computational tools employs MATLAB® for the purpose, and we believe students can benefit more by using mathematical software such as MATLAB and Mathematica® rather than system-specific software because of their limited scope.

For this author, writing the book has been a retirement project. He is indebted to Southern Methodist University and the Institute for the Study of Earth and Man for providing necessary resources and facilities even after his retirement. He acknowledges his gratitude to Professors Krishna Kavi and Robert Akl of the University of North Texas for contributing a chapter on the numerical analysis of queueing systems (in which the author's expertise is limited). Special acknowledgement of indebtedness is also made to the reviewers' comments, which have helped to improve the organization and contents of the book. The author also wishes to thank Professor N. Balakrishnan for recommending this book for inclusion in the *Statistics for Industry and Technology* series of Birkhäuser. Thanks are due to Professor Junfang Yu of the Department of Engineering Management, Information, and Systems of Southern Methodist University for using the prepublication copy of this book in his class and pointing out some of the typographical errors in it. Thanks are also due to Ms. Sheila Crain of the Department of Statistical Science for setting the manuscript in L^AT_EX with care and perseverance.

The author's wife, Girija, son Girish, and daughter Gouri have supported and encouraged him throughout his academic career. They deserve all the credit for his success.

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