Extremal Combinatorial Problems and Their Applications

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Extremal Combinatorial Problems and Their Applications

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To Our Mothers

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Preface to the English edition

The English version of this originally Russian book has been substantially supplemented by new material, and is almost 50 percent larger than the original. Part of the new material has been prepared in cooperation with colleagues. We express our deep gratitude to all of them: A. Klimov, A. Kostochka, I. Kan, I. Rival, V. Shmatkov, K. Rybnikov, A. Malykh, S. Sal'nikov, N. Sauer, A. Sidorenko, J. McIntosh, W. Kocay, C. Dodson, S. Radziszowski, V. Rodl, R. Wilson, and Gy. Katona.

In particular, Jack McIntosh offered the use of the word 'packability' for our very new Russian term '`vlozhimost'`'.

Our special thanks go to the translator. Having made attempts ourselves to translate combinatorial literature into Russian, we know how difficult it is to render correctly thoughts sometimes expressed in ponderous language or overloaded with meaning. However, we believe that even this paragraph has been successfully translated.

We thank `KLUWER Acad. Publ.`, who ventured to undertake this step, although we think that the risk has been substantially reduced by the superb coordination of the entire work done by Margaret Deignan, to whom we express our deepest gratitude.

Preparation of the English version of the book was partially supported for the second author by the Russian Foundation for fundamental research with its Algebraic Combinatorics grant ♯93 – 011 – 1442.

Introduction to the English edition

As far as we know, this is the first Russian book on general combinatorics that has been translated into English. In recent decades, the reverse process has occurred: most Western monographs, conference proceedings, and some collections of papers on combinatorics have been translated into Russian and published in large printruns.

In the post-War period, a vigorous development of combinatorial research has taken place in Russia on a two-fold basis: translated sources together with Russian books, conference proceedings and articles, and, finally, a journal specifically devoted to combinatorics. Russian combinatorists have been more fully informed than their Western colleagues. When reading our book one may get the feeling that we have devoted insufficient attention to foreign results. The Russian reader will know better: we exactly know its information sources and, candidly, have not feared the appearance of this translation. The book was written for Russian readers, with Russian objectives.

One such objective was to attract young people to the thematics of extremal problems and to combinatorics as a field of study. Therefore, in part, the book has the features of both a textbook and a reference handbook and can be read by mathematics students and beginning engineers. We have happily seen this objective realized in that the work of one of these students is presented in this English version by advances on the Frobenius problem.

Another objective is shown in our attempt to extend the extremal approach to solving a large class of problems including some previously regarded as exclusively algorithmic. Unfortunately, the problem '$\mathcal{P} = \mathcal{NP}$' has, from time to time, baffled not just theorists.

Related to this very objective is a third one (chronologically, the first): to broaden the freedom of choice of theoretical bases for modeling real phenomena in order to completely solve practical problems.

The real phenomenon which prompted this whole project is this: if a large number of problems (say, 108) are simultaneously solved by a computer, 'crowding' (fragmentation) of its memory occurs, sharply increasing both the total time and the separate time for solving each problem. This is sometimes of essential and even fundamental importance, for example, in the detection and servicing (destruction) of a set of fast-flying targets. And if their fly-up time (say, to Moscow) is five to eight minutes, then every computer second gained turns into entirely concrete reality.

The method is sufficiently universal, since every computer has a memory—even

the abacus, which, to this day, remains an unsurpassed configuration: it is simultaneously a carrier of memory, a processor, and a display, albeit necessarily with a person present.

Moscow V.I. Baranov
27 January 1993 B.S. Stechkin

Preface

This book is the result of the close cooperation between an engineer and a mathematician to develop methods for solving problems arising in the creation of automatic control systems. The main result of this cooperation is a combinatorial model, packability of number partitions, expounded in this book.

A study of packability of number partitions preceded the analysis of a number of practical problems arising in the design of efficient methods for controlling computer memory allocation, in the development of methods for the analysis of programming techniques for automatic control systems, etc. The choice of combinatorial methods was predetermined by the creation of the new, practically important, theme of extremal combinatorial problems on packability of number partitions. This combinatorial direction turned out to be useful not only in the formalization and solution of a number of problems in engineering, but also in solving a class of extremal problems on graphs.

The aim of this book is to let engineers and mathematicians become acquainted with the methods developed by the authors for solving a number of applied and mathematical problems. The material of the book is divided into 5 chapters.

Chapter 1 is a short guide to the necessary combinatorial notions. In particular, next to all elementary combinatorial schemes it explains a listing scheme developed by the authors, enabling us to unify the simplest combinatorial schemes.

Chapter 2 contains the basic mathematical results in the study of the packability of number partitions, and presents the, up till now, most complete summary of results in this direction. As illustration of the applicability of these results we note their relation to old weighing problems and other problem statements. We give as exercises some problems and assertions concerning the packability of number partitions.

Chapter 3 is devoted to getting acquainted with extremal problems on graphs and systems of sets. We show their relation with results on the packability of number partitions.

In Chapter 4 we study certain extremal problems in geometry, and apply the results of having solved them.

In Chapter 5 we give methods for using the results of solving extremal combinatorial problems concerning the packability of number partitions in the design of ACS. Here we give combinatorial models for studying processes to control the design of ACS and computer memory allocation. We demonstrate the applicability of the

packability theorem for computing the size of operation computer memory, and give definitions of a number of new concepts in engineering related to the application of methods of combinatorial analysis to the investigation of the performance of ACS. We propose a new method for estimating the efficiency of algorithms, characterized by extremal bounds.

The authors express their gratitude to all experts who helped in obtaining the results exposed in this book. In particular, O.V. Viskov, R.L. Graham, Ya. Demetrovich, Gy. Katona, Yu.V. Matiyasevich, S.G. Sal'nikov, and P. Erdös. The authors also thank A.F. Sidorenko, for supplementing the material of Chapter 3 by results concerning forbidden subgraphs and Ramsey numbers, and for taking part in writing the first two sections of Chapter 4. The authors express deep gratitude to A.A. Gushchin, V.K. Krivoshchekov, and A.A. Tsypkin for their great help in writing computer programs for obtaining the numerical results in Chapter 2.

Special thank of the authors goes to the reviewers, whose remarks not only helped in improving the book, but also influenced its structure.